

The architecture in Minkowski's space

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Abstract

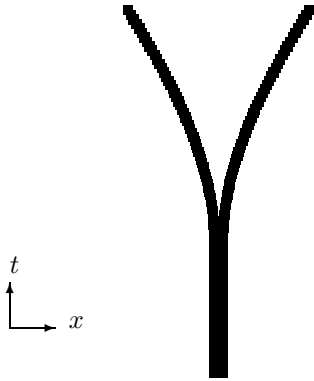
The purpose of this paper is to introduce a novel, scale covariant theory, possibly describing a very wide range of physical phenomena. In its greatest generality, this theory provides a rule governing the morphology — the ‘architecture’ — of space-time structures, viz., of admissible distributions of the energy-momentum tensor in Minkowski's space. This theory is shown to be compatible with the statistical predictions of quantum mechanics, namely, quantum mechanics could describe statistical attributes of ensembles of space-time structures generated by the proposed theory. As the nature of such ensembles cannot be deduced from the proposed single-structure theory, quantum mechanics must enjoys the status of a complementary, fundamental law of nature.

1 Introduction

The purpose of this paper is to introduce a novel, fully detailed theory, possibly describing a very wide range of physical phenomena. In its greatest generality, this theory provides a rule governing the morphology of space-time structures, viz., of admissible distributions of the energy-momentum (e-m) tensor in Minkowski's space. It describes the ‘architecture’ in Minkowski's space, irrespective of the presence of an observer (itself a space-time structure which may or may not influence the subject of his investigation).

This way of formulating a physical theory is not entirely new. In fact, every relativistic classical theory can be seen as a prescription for the set of admissible space-time structures. A typical example, to which we shall frequently refer, is classical electrodynamics of point charges. There, the e-m tensor is composed of a matter piece, supported on world-lines of charges, and an electromagnetic (EM) piece. The latter is focused around the world-lines of the charges (electrostatic component) and around the forward light-cones of points on ‘bent’ segments of those world-lines (radiative component). The physical content of classical electrodynamics is completely described by the set of such admissible structures, while the mathematical apparatus of Maxwell's equation and the Lorentz force equation, can be seen as a mere scaffold in the construction of individual structures. By tracing the e-m tensor, utilizing the tracelessness of the the EM piece, one can also define a scalar structure involving the matter piece only. Actually, it is only this latter scalar structure, describing the paths taken by all charges, which can truly be measured (our only knowledge about EM fields comes from their influence on the trajectories of charges).

Viewing a space-time structure as a single whole (rather than focusing on the local form of its scaffold) reveals a simple explanation for one of the most puzzling results of modern physics — ‘spooky’ correlations appearing between interacting particles. Consider, for concreteness, two nucleons escaping a nucleus in a radioactive decay. The corresponding space-time structure has the shape of a tree (see below).



The local morphologies of the two branches (even around points separated by a large space-like distance) are clearly not independent, as both branches eventually merge with one another at the trunk, and just as with physical trees, a large degree of coordination between the branches must exist for this to be possible. If each of the two nucleons eventually interacts with a polarimeter, the *global* structure of the tree is influenced by the orientations of both polarimeters. To each choice of orientations for the two polarimeters, there is a *different set* of trees which are compatible with that choice, accounting for the observed violation of Bell's inequalities in correlation experiments, which are derived on the premise that a *single* ensemble of trees is sampled, regardless of the orientations of the polarimeters (the motivation for Bell's assumption, which seems not only arbitrary, but also inconsistent from our perspective, is rooted in non relativistic thinking; see [1] for details). This simple example can be generalized to more complex structures, involving many particles, explaining other forms of collective behavior of matter which seem unintuitive from a nonrelativistic perspective, viewing matter as composed of individual particles with 'short term memory' at most.

A second benefit of expressing a physical theory as a set of admissible space-time structures is that it makes manifest the incompleteness of that very theory! Many physical questions deal with attributes of individual structures. For example, elementary particles or bound aggregates thereof are represented by stationary, isolated world-line like structures, from which attributes of a particle, such as its rest-energy, are to be read. But, as we shall see, there are other physical questions which are of statistical nature, requiring knowledge about *ensembles* of structures. The likelihood of realizing/encountering a particular structure in a given experimental/observational situation cannot be sought in the theory governing the shape of individual structures and therefore, an additional, fundamental law of nature, must be sought if we are to cope with such statistical issues. Quantum mechanics (QM), it is argued in this paper, is such a theory, dealing with some (and by no means all) statistical aspects of ensembles of space-time structures.

As of today, the only plausible candidate for the theory describing individual structures is classical electrodynamics. It compactly and accurately describes a huge range of physical phenomena (the very corpuscular nature of elementary charges being a much under appreciated one), and in certain cases it even reproduces the statistical results of QM, e.g. in

Coulomb scattering and in low energy Compton scattering. Nevertheless, classical electrodynamics, as is, cannot possibly be that theory. Strictly speaking, classical electrodynamics is not even a theory. The contribution of the self-EM field to the Lorentz force experienced by a point-charge, is ill defined. Moreover, classical electrodynamics does not support stable bound solutions (such as the trunk of the tree in the above example), and it is not entirely compatible with the statistical predictions of QM — not every result of the latter can be realized by an ensemble of solutions of the former (classical particles cannot diffract, for example).

It is therefore only natural to seek an upgraded, well defined ‘sibling’ of classical electrodynamics, which is compatible with the (immensely accurate) statistical predictions of QM. That sibling must also support bound, trunk like structures, and could potentially include the ‘strong force’, binding charges of the same sign, as one of its close-range manifestations (which, at any rate, must drastically deviate from classical electrodynamics at close-range, in order to resolve the self-force problem of the latter). Being a sibling of classical electrodynamics, this single-structure theory should also respect all the symmetries of the former, and in particular the hidden symmetry of *scale covariance*.

Roughly speaking, scale covariance expresses the idea that a fundamental physical theory must not contain a privileged length scale. Any preferred length scale appearing in nature, just like any preferred position, should be an attribute of a *particular* space-time structure, not of the *set* of all admissible structures, which must be invariant under scaling of space-time. Indeed, classical electrodynamics, with its point representation of charges, introduces no privileged length scale which is associated with a charge, and can be shown to be scale covariant. As we shall see, achieving scale covariance with extended charges is a lot more difficult, as no dimensionful parameter may be introduced into the theory from which the charge may inherit its typical scale. It calls for a method which is somewhat related to the renormalization procedure used in QED. One starts with a non scale covariant theory involving a dimensionful parameter which is ultimately removed, leaving a finite scale covariant theory. But unlike in QED, no arbitrary ‘infinite renormalization of parameters’ is involved in the process, making clear that isolation of the finite part of an expression is equivalent to the isolation of its scale covariant part.

The scale covariant theory describing individual space-time structures, presented in this paper, is a completion of a recently published theory by the current author, [4], dubbed extended charge dynamics (ECD). A more rigorous, albeit somewhat formal, presentation of ECD is given in the current paper, with most of the technical details appearing in a comprehensive appendices section. The reader who is also interested in the intuition behind the rather unusual mathematical structure of ECD, is referred to [4].

The lion’s share of this paper, however, is devoted to establishing the compatibility of ECD with QM. As explained before, assuming there exists a theory describing individual structures (allegedly ECD), the theory dealing with statistical aspects of ensembles of structures (allegedly QM) cannot be derived from the former. Nevertheless, severe mutual constraints do exist between the two and section 3 demonstrates in representative cases that they are satisfied in the case of ECD and QM. It is further shown that the results of classical

electrodynamics are reproduced by ECD in the former's domain of validity. Finally, the radical implications to gravity of generalizing ECD to curved space-time, are briefly discussed in section 4.

2 Extended Charge Dynamics

A note about dimensions. The custom of attaching a ‘dimension’ (in the usual sense of mass, length, mass/length etc.) to constants and variables appearing in the equations of physics, not only does it lead to awkward combinations (e.g. elements in some abstract algebra expressed in kilos...) but, in fact, it is unnecessary. Any physically meaningful statement involves only pure (real) numbers, expressing the ratio between two quantities of the same ‘dimensionality’. Accordingly, throughout this paper, functions defined on Minkowski's space-time, \mathbb{M} , have their values in the relevant abstract mathematical space, viz. no ‘dimension’ is attached to those objects. In particular, points in space-time are indexed by four labels — real numbers. The labeling convention of the (affine) space-time grid is chosen so that the speed of light equals 1 everywhere, and in all directions. This determines the labeling of space-time up to a Poincaré transformation *and* an arbitrary scale factor $\lambda > 0$. This arbitrariness can always be compensated by suitably rescaling all objects, viz., constants and functions defined on space-time, $\Omega \mapsto \lambda^{-P_\Omega} \Omega$, and P_Ω is what is customarily referred to as the ‘length dimension’ (e.g. in QFT), or simply dimension of object Ω . Physical ratios between two objects of the same dimensionality are clearly independent of λ .

2.1 Manifestly scale covariant classical electrodynamics

The affinity of ECD to classical electrodynamics in terms of symmetries and conservation laws, warrants an unorthodox formulation of the latter in a form which is manifestly scale covariant. There are two components in classical electrodynamics of n interacting charges. One is the Lorentz force, governing the motion of a charge in a fixed EM field

$$\ddot{\gamma}^\mu = q F^\mu{}_\nu \dot{\gamma}^\nu, \quad (1)$$

with $\gamma(s) \equiv \gamma_s : \mathbb{R} \mapsto \mathbb{M}$ the world line of a charge, parametrized by the Lorentz scalar s , q a coupling constant and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the antisymmetric Faraday tensor. Multiplying both sides by $\dot{\gamma}_\mu$ and using the antisymmetry of F , we get that $\frac{d}{ds} \dot{\gamma}^2 = 0$, hence $\dot{\gamma}^2$ is conserved by the s -evolution. This is a direct consequence of the s -independence of the Lorentz force, and can also be expressed as the conservation of a ‘mass-squared current’

$$b(x) = \int_{-\infty}^{\infty} ds \delta^{(4)}(x - \gamma_s) \dot{\gamma}_s^2 \dot{\gamma}_s. \quad (2)$$

Defining $m = (\int d^3\mathbf{x} b^0)^{1/2} \equiv \sqrt{\dot{\gamma}^2} \equiv \frac{d\tau}{ds}$ with $\tau = \int^s \sqrt{(\dot{\gamma})^2}$ the proper-time, equation (1) takes the familiar form

$$m \ddot{x}^\mu = q F^\mu{}_\nu \dot{x}^\nu, \quad (3)$$

with $x(\tau) = \gamma(s(\tau))$ above standing for the same world-line parametrized by proper-time. We see that the (conserved) effective mass m emerges as a constant of motion associated with a particular solution rather than entering the equations as a fixed parameter. Equation (1), however, is more general than (3), and supports solutions conserving a negative $\dot{\gamma}^2$ (tachyons — irrespective of their questionable reality) as well as a vanishing $\dot{\gamma}^2$.¹

The second ingredient of classical electrodynamics is Maxwell's inhomogeneous equations, prescribing an EM potential given the world-lines of all charges

$$\partial_\nu F^{\nu\mu} \equiv \square A^\mu - \partial^\mu(\partial \cdot A) = \sum_{k=1}^n k j^\mu, \quad (4)$$

with

$$^k j(x) = q \int_{-\infty}^{\infty} ds \delta^{(4)}(x - \gamma_s) \dot{\gamma}_s^k \quad (5)$$

the electric current associated with charge k , which is conserved,

$$\partial_\mu j^\mu = q \int_{-\infty}^{\infty} ds \partial_\mu \delta^{(4)}(x - \gamma_s) \dot{\gamma}_s^\mu = -q \int_{-\infty}^{\infty} ds \partial_s \delta^{(4)}(x - \gamma_s) = 0.$$

The self-force problem of classical electrodynamics, to which we shall return in section 2.3, refers to the fact that the EM field generated by (4) diverges everywhere on the world line, $\bar{\gamma} \equiv \cup_s \gamma_s$, traced by γ , rendering the Lorentz force (1) ill defined (Another troubling aspect of the self-force problem is the divergence of formally conserved quantities such as energy and momenta.)

The above unorthodox formulation of classical electrodynamics highlights its *scale covariance*, meaning that the scaled variables

$$A'(x) = \lambda^{-1} A(\lambda^{-1} x), \quad \gamma'(s) = \lambda \gamma(\lambda^{-2} s), \quad (6)$$

also solve (1)² and (4), *without scaling of any parameter*. Like the Poincaré symmetry, the scaling symmetry (6) admits both an active and a passive interpretation. In the active sense, it relates between different solutions of the theory, for a given labeling (unit-length convention) of space-time, from which one can read the representation under which each variable, and products thereof, transform in a scale transformation — its ‘scaling dimension’: $[x] = [\gamma] = 1$; $[s] = 2$; $[A] = [m] = -1$; $[j] = -3$ and, by definition, $[q] = 0$.

In the passive interpretation, the symmetry (6) prescribes how one must rescale A (more generally, functions defined on space-time) when relabeling (scaling the unit-length) space-time. In this sense, A can be measured in units of length, and its scaling dimension may just as well be named its length dimension, or simply dimension.

¹Classical dynamics of a massless charge is commonly defined by setting $m = 0$ in (3), which is not the same as using (1) subject to the initial condition $\dot{\gamma}^2 = 0$

²An addition of the Lorentz-Dirac radiation reaction force, written in our convention as $\frac{2}{3}q^2 \left(\frac{\ddot{\gamma}}{\dot{\gamma}^2} - \frac{\dot{\gamma} \cdot \ddot{\gamma} \dot{\gamma}}{(\dot{\gamma}^2)^2} \right)$, still preserves the symmetry (6).

The simplicity in which scale covariance emerges in classical electrodynamics is due to the representation of a charge by a mathematical point, obviously invariant under scaling of space-time. As we shall see, achieving scale covariance with extended charges is a lot more difficult, as no dimensionful parameter may be introduced into the theory from which the charge may inherit its typical scale.

Associated with symmetry (6) is an interesting conserved ‘dilatation current’

$$\xi^\nu = p^{\nu\mu} x_\mu - \sum_{k=1}^n \int ds \delta^{(4)}(x - {}^k\gamma_s) s \, {}^k\dot{\gamma}_s^2 \, {}^k\dot{\gamma}_s^\nu, \quad (7)$$

with

$$p^{\nu\mu}(x) = \frac{1}{4} g^{\nu\mu} F^2 + F^{\nu\rho} F_\rho{}^\mu + \sum_{k=1}^n {}^k m^{\nu\mu}, \quad (8)$$

the (formally) conserved energy-momentum (e-m) tensor associated with translation covariance, and

$$m^{\nu\mu} = \int_{-\infty}^{\infty} ds \, \dot{\gamma}^\nu \dot{\gamma}^\mu \delta^{(4)}(x - \gamma_s), \quad (9)$$

the ‘matter’ e-m tensor, formally satisfying

$$\partial_\nu m^{\nu\mu} = F^{\mu\nu} j_\nu, \quad (10)$$

$$\begin{aligned} \partial_\nu m^{\nu\mu} &= \int ds \, \dot{\gamma}^\nu \dot{\gamma}^\mu \partial_\nu \delta^{(4)}(x - \gamma_s) = - \int ds \, \dot{\gamma}^\mu \partial_s \delta^{(4)}(x - \gamma_s) \\ &= \int ds \, \ddot{\gamma}^\mu \delta^{(4)}(x - \gamma_s) = \int ds \, q F^{\mu\nu} \dot{\gamma}_\nu \delta^{(4)}(x - \gamma_s) = F^{\mu\nu} j_\nu. \end{aligned}$$

Note that the conserved dilatation charge, $\int d^3\mathbf{x} \xi^0$, depends on the choice of origin for both space-time, and the n parameterizations of ${}^k\gamma$, and is therefore difficult to interpret.

2.2 Extended Charge Dynamics

In a nutshell, the transition from classical electrodynamics to ECD, involves two modifications. The first grants the electric current (5) a nonsingular support in a way respecting all the symmetries of classical electrodynamics — scale covariance in particular. To this end we add to the representation of each charge an auxiliary complex (more generally spinor valued; see appendix D) ‘wave-function’ ${}^k\phi(x, s) : \mathbb{M} \times \mathbb{R} \mapsto \mathbb{C}$, and modify the current (5) to read

$${}^k j^\mu(x) = \int_{-\infty}^{\infty} ds \, \frac{iq}{2} \left[{}^k\phi \left(D^\mu {}^k\phi \right)^* - {}^k\phi^* D^\mu {}^k\phi \right] \equiv \int_{-\infty}^{\infty} ds \, q \operatorname{Im} \left[{}^k\phi^* D^\mu {}^k\phi \right], \quad (11)$$

with

$$D_\mu = \bar{h} \partial_\mu - iq A_\mu \quad (12)$$

the gauge covariant derivative and \bar{h} some real dimensionless ‘quantum parameter’, not to be confused with \hbar . Note the similar structure of (11) and (5). In (5) it is the trace in Minkowski’s space of a singular vector-valued distribution, $\delta^{(4)}(x - {}^k\gamma_s) {}^k\gamma_s^\mu$, generating the current, whereas in (11), the corresponding distribution is $\text{Im } {}^k\phi^* D^\mu {}^k\phi$, and need not be singular. Despite this similarity between the ECD current (11) and the classical current (5), there is a striking difference between the two: the EM potential A enters the definition of the current (through D ’s dependence on it) which, in turn, depends on *all* charges. It will be demonstrated how this interdependence, along with an implicit dependence of ϕ on A , described next, leads to quantum mechanical ‘entanglement’.

Summarizing, each ECD charge is now represented by a pair $\{\phi, \gamma\}$ but, of course, ϕ is not independent of γ , as described next.

The central ECD system. The second component of ECD is the *central ECD system* — the counterpart of the Lorentz force equation (1) — prescribing the set of permissible pairs $\{\phi, \gamma\}$ for a given A . Unlike (1), however, the ECD counterpart also applies to chargeless particles, viz. particles with a vanishing monopole. This system is composed of two coupled equations. The first reads

$$\begin{aligned} \phi(x, s) &= -2\pi^2 \bar{h}^2 \epsilon i \int_{-\infty}^{s-\epsilon} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \\ &\quad + 2\pi^2 \bar{h}^2 \epsilon i \int_{s+\epsilon}^{\infty} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \\ &\equiv -2\pi^2 \bar{h}^2 \epsilon i \int_{-\infty}^{\infty} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \mathcal{U}(\epsilon; s - s'), \end{aligned} \quad (13)$$

$$\text{with} \quad \mathcal{U}(\epsilon; \sigma) = \theta(\sigma - \epsilon) - \theta(-\sigma - \epsilon),$$

and the second equation is

$$\partial_x |\phi(x, s)|^2 \big|_{x=\gamma_s} \equiv \partial_x |\phi(\gamma_s, s)|^2 = 0. \quad (14)$$

Above, $G(x, x'; s)$ is the *propagator* of a proper-time Schrödinger equation (also known as a five dimensional Schrödinger equation, or Stueckelberg’s equation),

$$\left[i\bar{h}\partial_s - \mathcal{H}(x) \right] G(x, x'; s) = 0, \quad \text{with } \mathcal{H} = -\frac{1}{2}D^2, \quad (15)$$

satisfying the initial condition (in the distributional sense),

$$G(x, x'; s) \xrightarrow{s \rightarrow 0} \delta^{(4)}(x - x'). \quad (16)$$

Finally, ϵ is a parameter of dimension 2, ultimately taken to zero (thereby eliminating the single dimensionful parameter of ECD).

Both the central ECD system (13), (14), and the ECD current (11), involve a delicate $\epsilon \rightarrow 0$ limit which is discussed in detail in appendices A and C. Behind the rather involved mathematics lies a magnificently simple conclusion: All ECD currents, the electric current

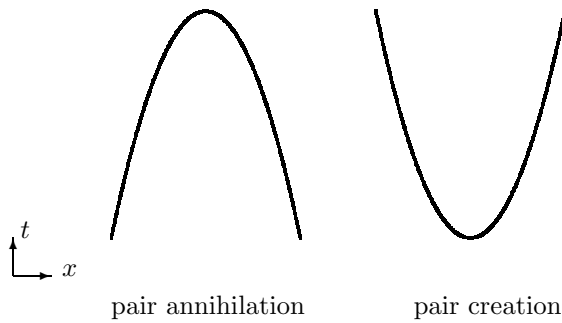
(11) and the ECD counterpart of the classical energy-momentum tensor (9) in particular, become (integrably) singular on the world-line $\bar{\gamma} \equiv \cup_s \gamma_s$. The central ECD system is nothing but the condition that no electric charge nor energy-momentum leak into those ‘world-sinks’ on $\bar{\gamma}$.

Carefully applying Noether’s theorem, both charge and energy-momentum conservation, can be shown to follow from continuous symmetries of ECD, (see appendix C). The converse, however, is not true, namely, not every continuous symmetry of ECD leads to a conservation law, due to a possible leakage of the corresponding charge to sinks on ${}^k\bar{\gamma}$. The counterparts of the ‘mass-squared current’, (2), associated with s -translation invariance, as well as the counterpart of (7) corresponding to scale covariance, fall into this category (see appendix C).

Antiparticles. But perhaps a greater deviation of ECD from classical notions is manifested in the possibility of γ to ‘turn back in time’. As it only marks the center of an ECD current, γ is not constrained to a fixed mass shell as in classical electrodynamics, irrespective of the above mentioned leakage of mass. Combined with a ‘CPT’ symmetry of ECD, the notion of particle-antiparticle creation/annihilation is inevitable. Indeed, ECD can be shown to be invariant under a ‘CPT’ transformation

$$\begin{aligned} A(x) &\mapsto -A(-x), \quad \gamma(s) \mapsto -\gamma(-s) \quad \phi(x, s) \mapsto \phi^*(-x, -s) \\ &\Rightarrow j(x) \mapsto -j(-x). \end{aligned} \tag{17}$$

In fact, scalar ECD, as well as classical electrodynamics³, enjoys an even larger symmetry group, C: $A(x) \mapsto -A(x)$, $j(x) \mapsto -j(x)$; and PT: $A(x) \mapsto A(-x)$, $j(x) \mapsto j(-x)$. However, the spin- $\frac{1}{2}$ ECD, presented in appendix D, enjoy the CPT symmetry only. This symmetry has some remarkable consequences. First, it implies that our naive notion of time-reversal — ‘running the movies backward’ — is not a symmetry of micro-physics. Secondly, it predicts the existence of an antiparticle for each particle (viz., a bound solution of one or more elementary ECD charges) of opposite charge and equal self-energy. Pair creation/annihilation may then have a simple geometrical interpretation when γ ‘reverses its direction in time’ (see picture).



³Maxwell’s equations and the Lorentz force are also symmetric under under T: $\gamma(t) \mapsto \gamma(-t)$, $\mathbf{E}(\mathbf{x}, t) \mapsto \mathbf{E}(\mathbf{x}, -t)$, $\mathbf{B}(\mathbf{x}, t) \mapsto -\mathbf{B}(\mathbf{x}, -t)$, and under P: $\gamma(t) \mapsto -\gamma(t)$, $\mathbf{E}(\mathbf{x}, t) \mapsto \mathbf{E}(-\mathbf{x}, t)$, $\mathbf{B}(\mathbf{x}, t) \mapsto -\mathbf{B}(-\mathbf{x}, t)$. However, if one includes in the definition of classical electrodynamics, a definite Green’s function, other than the half-advanced-plus-half-retarded-Lienard-Wiechert-potential, then T is no longer a symmetry.

As a particle and its antiparticle have opposite signs for both their electric charges, and their mass-squared charges (expression (95), the counterpart of the classical (2)), such annihilation/creation scenarios respect the conservation laws of both electric and mass-squared charges. The energy of a particle, however, equals that of its antiparticle. In such annihilation processes, either EM radiation must be released or else a different pair (pairs) must be created, in order to respect energy conservation.

2.3 The self consistent potential

As in classical electrodynamics, so also in ECD, the EM potential A must satisfy a self consistent ‘loop’:

- (a) Start with A and n pairs $\{^k\phi, ^k\gamma_s\}$ satisfying the central ECD system (50), (51) (or the Lorentz force equation, (1), in classical electrodynamics);
- (b) From these, using (11) (or (5) in classical electrodynamics), compute n currents $^k j$ ’s, plug them into the r.h.s. of (4) and, finally,
- (c) verify that the the l.h.s. agrees with the original A .

The common loop notwithstanding, two important differences should be noted. First, in classical electrodynamics, the loop is only formal, due to the self-force problem — the ill defined Lorentz self-force at the position of a point-charge. In ECD, on the other hand, only $\phi(x, s)$ needs to be differentiable on $x \in \bar{\gamma}$. This condition easily tolerates discontinuities of the EM field on $\bar{\gamma}$ (which, in fact, exist) freeing ECD, as is, from the self-force problem.

The second difference in the role played by the above loop is that, in ECD, *the very existence of an ECD charge is due to a solution for the loop*. That is, a non vanishing A must be found even for a single static charge in an otherwise void universe — different such solutions naturally corresponding to different elementary particles. This is a nontrivial requirement, possibly leading to constraints on the nature of fundamental ECD charges. Charge quantization is one such possibility, as the magnitude of the total charge of a solution is invariant under the full symmetry group of ECD. Another possibility is that, as in other eigenvalue problems, only for certain values of the ECD parameters, viz. \bar{h} , q , and g (for spin- $\frac{1}{2}$ ECD), does there exist a solution.

3 Qualitative discussion of ECD

The ECD formalism presented in the previous section has a rather unusual structure. Explicit solutions, relevant to physically interesting cases, are difficult to solve, apparently necessitating an extensive use of numerical calculations. However, the stage is completely set for such detailed analysis. Isolated, self consistent ECD solutions or bound states of any number of them, can be sought, possibly (and most desirably in the author’s opinion) showing that all elementary particles are just different solutions of the same set of equations (significantly reducing the number of tunable constants). The effective mass and binding energies of such solutions can be computed using the expression for the energy momentum tensor derived in the appendices; detailed internal structures of such particles can be an-

alyzed, possibly suggesting novel methods of ‘cracking’ (or fusing) subatomic particles. In short, one can potentially have a clear, scale covariant ontology, based on interacting ECD particles alone, rendering additional forces and particles superfluous. ECD, then, is clearly not merely an interpretation of QM, but rather a complementary theory with independent testable predictions, just as envisioned by Einstein [5].

To motivate such an endeavor, this section argues the case for the compatibility of ECD with current, well tested, physics. ECD, essentially retaining the ontology of classical electrodynamics, is apparently susceptible to the same objections and ‘no go theorems’ standing in the way of other hidden variables models. In particular, since the EM field is just the classical Maxwellian field, one may rightfully wonder where does the ‘photon’ come from? We shall demonstrate to the contrary, that in a wide range of cases in which the statistical predictions of QM clearly cannot be realized by an ensemble of classical solutions, the unique features of ECD could render possible such a realization by an ensemble of ECD solutions, resulting in a rather prosaic ‘demystification’ of QM — the photon included.

For simplicity, only the scalar case is analyzed. The spin of an ECD particle merely labels different ways of covariantly obtaining extended space-time structures — ordinary tensor-valued distributions. An example of spin- $\frac{1}{2}$ ECD is covered in appendix D.

3.1 Single-body ECD

Single-body ECD deals with the ECD equations of a single particle in the presence of an external EM potential, where the use of the term ‘particle’, rather than charge, reflects the possibility for an elementary ECD solution to have a vanishing monopole. Specifically, we assume the existence of an external potential, A_{ext} , satisfying Maxwell’s equations (4) for some fixed current, j_{ext} , generated by the rest of the particles in the universe, assumed independent of the particle in question. This is clearly a simplification of the real situation to which we return in section 3.2, dealing with many-body ECD. Next, we ‘feed’ $A_{\text{ext}} + A_{\text{sel}}$ into the self consistent loop of section 2.3, solving ϕ in the presence of $A_{\text{ext}} + A_{\text{sel}}$, and close the loop by requiring from the self potential to satisfy

$$\square A_{\text{sel}}{}^\mu - \partial^\mu (\partial \cdot A_{\text{sel}}) = j^{\text{r}}{}^\mu, \quad (18)$$

with j^{r} computed from ϕ and the combined potential $A_{\text{ext}} + A_{\text{sel}}$, and the ‘r’ superscript standing for ‘regularized’ — as explained in appendix A.

Modulo the self-force problem, the corresponding classical problem trivializes to finding solutions of (1) in the the presence of F_{ext} . While the exact ECD solution, explicitly incorporating self interaction effects, is much more complicated, its energy-momentum balance follows quite similar lines. In appendix C, a relation, (90), is derived, formally identical to its classical counterpart (10), relating the energy momentum tensor, m , associated with a particle, to its conserved electric current j (omitting the regularization label $^{\text{r}}$)

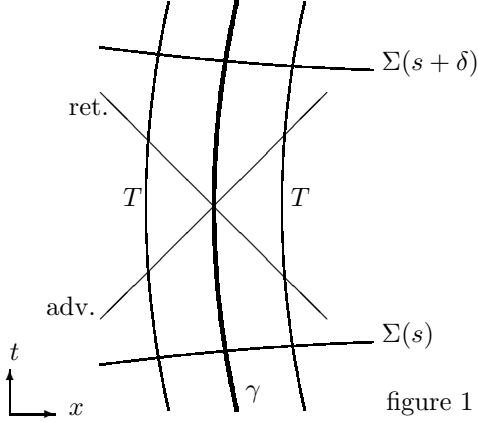
$$\partial_\nu m^{\nu\mu} = F^{\mu\nu} j_\nu \equiv (F_{\text{ext}}{}^{\mu\nu} + F_{\text{sel}}{}^{\mu\nu}) j_\nu, \quad (19)$$

where F_{sel} is the self-field derived from A_{sel} via (18). Let $\Sigma(s)$ be a one-parameter family of non intersecting time-like surfaces, each intersecting $\bar{\gamma}$ at γ_s , C a four-cylinder containing $\bar{\gamma}$

and $p^\mu(s)$ the corresponding four-momenta

$$p^\mu = \int_{\Sigma(s) \cap C} d\Sigma_\nu m^{\nu\mu}, \quad (20)$$

where $d\Sigma$ is the Lorentz covariant directed surface element, orthogonal to $\Sigma(s)$. Let also $C(s, \delta) \in C$ be the volume enclosed between $\Sigma(s)$ and $\Sigma(s + \delta)$, and $T(s, \delta)$ its space-like boundary (see figure 1 for a 1 + 1 counterpart).



Integrating (19) over $C(s, \delta)$, and applying Stoke's theorem to the l.h.s., we get

$$p^\mu(s + \delta) - p^\mu(s) + \int_T dT_\nu m^{\nu\mu} = \int_{C(s, \delta)} d^4x (F_{\text{ext}}^{\mu\nu} + F_{\text{sel}}^{\mu\nu}) j_\nu. \quad (21)$$

with dT the outward pointing directed surface element on T . For a point charge with m and j given by (10) and (5) resp., the term $\int_T dT_\nu m^{\nu\mu}$ vanishes, $p = \dot{\gamma}$, and upon taking the limit $\delta \rightarrow 0$ and dividing by δ , (21) formally becomes just the Lorentz force equation (1) with $F = F_{\text{ext}} + F_{\text{sel}}$. As noted before, however, the self force is ill-defined in the classical case (hence the reservation implied in ‘formally’). In moving from a singular electric current to an extended one, the first benefit is that now the self-force appearing in (21) is well defined. For a static charge, for example, the only non vanishing component of the electric current is $j^0(\mathbf{x})$ from which the purely electrostatic F_{sel} inherits its spherical symmetry, leading to a vanishing self force. The simplest complication of the static case, then, is when the currents retain an approximate spherical symmetry (in the rest frame of γ) and F_{sel} is approximately a non radiating spherical electrostatic field, contributing a negligible self-force only. Under this assumption we can write $p = \alpha \dot{\gamma}$ with α some positive constant, and

$$\lim_{\delta \rightarrow 0} \delta^{-1} \int_{C(s, \delta)} d^4x F_{\text{ext}}^{\mu\nu} j_\nu = Q \langle F^{\mu\nu} \rangle_s \dot{\gamma}_\nu, \quad (22)$$

with $Q = \int_{\Sigma(s)} d\Sigma \cdot j$ the s -independent electric charge (here we assume that $\Sigma(s) \cap C$ supports the lion's share of the charge) and $\langle F^{\mu\nu} \rangle_s$ is the average field in $\Sigma(s)$, weighted by the normalized charge density. (The above equalities are most conveniently established in

the rest-frame of γ where j^0 and m^{00} are the only non vanishing components, $\Sigma(s)$ is taken to be $x^0 = \gamma_s^0$ three-space, the Lorentz force density is purely electrostatic, and $dT_0 = 0 \Rightarrow dT_\nu m^{\nu\mu} = 0$.) Equation (21) then leads to

$$\alpha \ddot{\gamma}^\mu = Q \langle F^{\mu\nu} \rangle_s \dot{\gamma}_\nu, \quad (23)$$

and the constant α is identified with $\sqrt{p^2/\dot{\gamma}^2}$, where p^2 is the Lorentz invariant rest-energy of the charge. We therefore reach the important conclusion: *Whenever an ECD charge maintains an approximate spherical symmetry, its dynamics must be classical.*

It is instructive to compare the above treatment of an ECD charge with Lorentz's modeling of the electron as a rigid, uniformly charged sphere, enabling him to obtain a well defined expression for the self-force, without going through a fishy mass-renormalization procedure (as in later treatments, preserving the point structure of the charge). As a relativistic rigid extended body is a meaningless concept, Lorentz's sphere model is valid at most for a sufficiently uniform motion — the larger the sphere, the more uniform the motion must be. A rapidly varying external field on the scale of the sphere therefore signals the breakdown of Lorentz's self-force analysis. Likewise, a non uniformly moving ECD particle cannot maintain an exactly spherical charge distribution in the rest frame of every point along γ , and a rapidly varying γ on the scale of the ball holding the lion's share of the charge, dubbed the *core*, needs not even resemble a classical path. In this respect, ECD can be seen as a fully covariant extension of Lorentz's analysis of the self-force. It is argued below that, in principle, all of QM can be traced to the breakdown of the spherical core approximation.

Assuming ECD indeed governs the microscopic world, the above spherical core model explains at once the reductions of the QED Klein-Nishina formula for the cross section in Compton scattering, to the classical Thompson formula, at wavelengths greatly exceeding the electron's Compton length, and sets the Compton length as the order of magnitude of the core. Indeed the Thompson formula is obtained by simple averaging over the radiation produced by point charges oscillating in an external plane wave. For wavelengths much longer than the scale of the core, we have in (23) $\langle F \rangle_s \approx F(\gamma_s)$, and point dynamics is reproduced. The spherical core approximation further accounts for another conspicuous coincidence — the agreement between the nonrelativistic classical and quantum cross sections for Coulomb scattering, both given by the \hbar -independent Rutherford formula. The fact that the Coulomb potential (being an electrostatic potential) is a harmonic function, implies that the potential of a spherically symmetric core in it, equals that of the center of the core, viz. that of γ , hence no finite-core-size corrections to point dynamics are observed for this special potential.

3.1.1 The breakdown of the spherical core

Equation (19) and its integral form (21), somewhat artificially divide the change in the momentum of a particle into a work of the Lorentz force, plus a 'radiative' contribution, $\int_T dT_\nu m^{\nu\mu}$, of the associated e-m density m . A more symmetric treatment of 'matter' and the EM field is provided by the conservation of the ECD counterpart of (8), $p = \Theta + \sum_k {}^k m$ (see appendix C.1) where Θ is the canonical EM energy-momentum tensor (93). Applying

Stoke's theorem to $\partial p = 0$, and using the same construction as in figure 1, we get

$$p^\mu(s + \delta) - p^\mu(s) = - \int_T dT_\nu p^{\nu\mu}, \quad (24)$$

with

$$p^\mu = \int_{\Sigma(s) \cap C} d\Sigma_\nu p^{\nu\mu}, \quad (25)$$

the total four-momentum content of $\Sigma(s) \cap C$. Although $p^{\mu\nu}$ is due to *all* particles in the system, in the vicinity of a sufficiently isolated particle k' , p is dominated by ${}^{k'}m$ and the self field generated by ${}^{k'}j$. This all leads to the conclusion that the conservation of energy and momentum associated with an isolated particle (EM e-m included) can only be breached by an energy-momentum flux penetrating T . This flux is composed of the classical Poynting vector, plus a ‘quantum’ piece associated with ${}^{k'}m$. This ‘electro-weak’ division, nonetheless, is entirely artificial, as ${}^{k'}m$ also depends on A both explicitly and implicitly (via ϕ). Moreover, whenever the core breaks down, producing such matter e-m flux over T , a corresponding electric flux also forms, which locally modifies the Pointing flux (note that this ‘radiative component’ of j may be negligible in terms of charge capacity, and still generate a strong EM field if it strongly fluctuates).

3.1.2 The necessity for advanced fields

Summarizing our findings regarding a sufficiently isolated particle, the integral of the e-m flux, $\int_T dT_\nu p^{\nu\mu}$, does not vanish only if γ is non uniform. This, of course, is a standard result of classical electrodynamics, the quantification of which leads to the celebrated Lorentz-Dirac equation (but not before ‘renormalizing’ the mass of the charge — a rather contrived procedure whose sole motivation is to render the result non trivial), and can be directly derived from the the expression for the Lienard-Wiechert potential generated by a moving charge

$$A_{\text{adv}}^{\text{ret}}(x) = q \int ds \delta[(x - \gamma_s)^2] \dot{\gamma}_s \theta(x^0 \mp \gamma_s^0). \quad (26)$$

The advanced and retarded potentials are the traces of densities, $\delta[(x - \gamma_s)^2] \dot{\gamma}_s \theta(x^0 \mp \gamma_s^0)$, supported on the light-cone of γ_s (schematically depicted by adv. and ret. in figure 1). These fields are then further processed, yielding a Θ which can easily be shown to contain a radiative component (viz. dropping as r^{-2} from the charge) at a point x only if γ is non uniformly moving in the neighborhood of points lying on the intersections of the light-cone of x with $\bar{\gamma}$.

The ECD counterpart of (26) can be expected to read

$$A_{\text{adv}}^{\text{ret}}(x) = q \int ds \left[\int d^4y \delta[(x - y)^2] \theta(x^0 \mp y^0) \lim_{\epsilon \rightarrow 0} J^\epsilon(s, y) \right], \quad (27)$$

with J^ϵ the regularized⁴ density $\text{Im } \phi^* D \phi$. Indeed, A appearing in (27) solves Maxwell's

⁴A regularized density is one which is removed of bilinears in ϕ^s , and the $\epsilon \rightarrow 0$ limit is to be understood in the distributional sense. See appendix A for details.

equations (4) with j^r as a source, and upon substituting in (27) the corresponding classical density $J^r(s, y) \mapsto \delta^{(4)}(y - \gamma_s) \dot{\gamma}_s$, (26) is reproduced. Equation (27), however, ignores a crucial difference between the above densities. Unlike its classical counterpart, J^r depends on A . Equation (27), unlike (26), is therefore not a prescription for $A_{\text{adv}}^{\text{ret}}$ but rather an equation for it. A solution in which only A_{adv} or A_{ret} enter J^r may not (and probably does not) exist. The ‘correct’ radiation field, containing both advanced and retarded components, so to speak, is therefore dictated by the specifics of the radiation process. In classical electrodynamics, in contrast, a solution of Maxwell’s equation (4) is defined only up to a solution of the homogeneous equation $\partial_\nu F^{\nu\mu} = 0$. For this reason, the motivation for the (almost universal) choice of the retarded potential is not in the equations proper, but rather in the desire to conform with observations concerning *large scale* radiation phenomena, involving huge numbers of particles (See more in section 3.2.2). We are led, then, to the important conclusion that in ECD, advanced EM flux, combined with a corresponding advanced mechanical flux associated with m , must be considered. We shall see in 3.2.1 apparent experimental signatures left by such advanced effects.

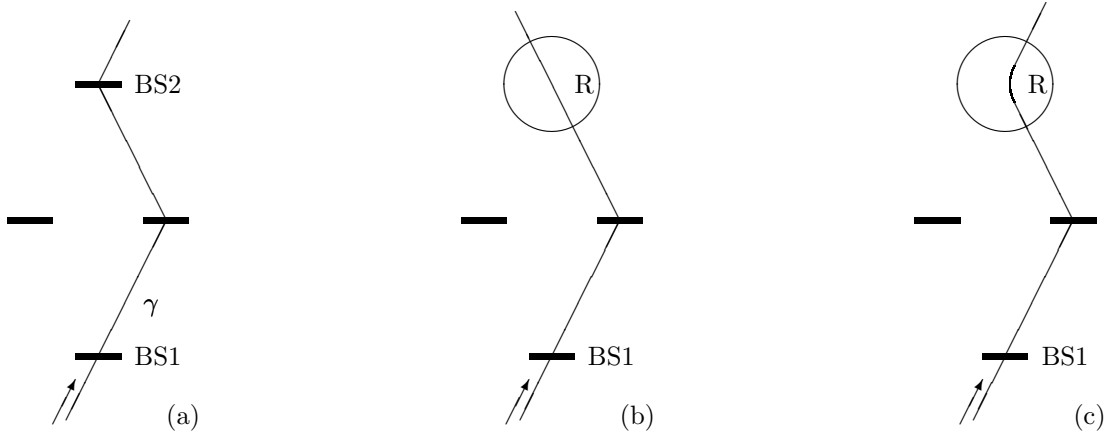
3.1.3 The aether and its manifestations

The picture emerging from the previous section is that of a single conserved e-m field, $p^{\mu\nu}(x)$, organically integrating, or fusing, radiation and matter. The distribution of p in \mathbb{M} is highly nonuniform, with the lion’s share of the charges concentrated in small three-cylinders centered around ${}^k\bar{\gamma}$ (and integrably singular there; see appendix C). Time-like cross sections of these tubes were dubbed ‘cores’ of particles, and it is these cores which are responsible for the thin traces forming in particle detectors and, in general, to the illusion of point particles. Amidst the densely charged three-cylinders resides a (mostly) weak, locally conserved p , smoothly merging with the dense cylinders. This inter-particle p which, in the absence of a better name shall be referred to as the *aether*, cannot be decisively attributed to any single particle as A , entering every term in p , is generated by the combined current of *all* the particles. Nevertheless, as we have seen, its dynamics at a point is strongly correlated with that of neighboring particles.

Consider now a scattering experiment in which a neutral particle passes through a small aperture in a screen. As only the collective p — cores plus aether — is conserved, there is a mutual influence between the particle and the (cores composing the) screen, mediated by the aether. This interaction, however, is extremely small in ordinary laboratory experiments, and it is only via the amplification brought about by the huge distance between the aperture (more generally — a scatterer) and the detection screen, that minute corrections to the classical cross section are detectable. One can also understand why attempts to measure the position of the particle during its flight, by shining light on it, destroy the delicate interference pattern. This is because the external EM field applied to the particle, interacts with the core. This is essentially a classical interaction, coming with a variability which greatly surpasses the feeble quantum corrections to classical paths, responsible for the fringes in the first place.

3.1.4 Interferometers

We saw how the huge distance between the scatterer and the detection screen can be used to amplify the small aether-induced deviations of the cores from classical paths. Another amplification technique, implemented in neutron interferometers, relies on the ability of chaotic systems to amplify small perturbations. In a Mach-Zehnder configuration, (a), the beam-splitters (BS) and mirrors are crystals of macroscopic thickness, forming a huge lattice of scatterers in which a particle undergoes multiple scatterings before exiting.



Even at the classical level, the dynamics in such a maze is highly chaotic, meaning, in particular, that the standard procedure of averaging over the impact parameter in order to obtain the scattering cross section, is utterly meaningless⁵.

⁵One of the first lessons in undergraduate physics teaches you how to derive the scattering cross section in a classical scattering experiment. Given the form of the potential and the equations of motion, one first constructs the scattering map $S : b \mapsto \Omega$, where b is the impact parameter of an incident particle and Ω the scattering angle of the particle. One then takes a suitable ensemble of solutions, characterized by the distribution of its impact parameter (typically a uniform distribution) from which the scattering cross section is obtained via S .

What is not taught in those lessons is that this method of obtaining the scattering cross section is consistent only for potentials for which the dynamics of the scattered particle is integrable. When applied to so called chaotic targets, S is no longer continuous, generating a cross section which is a fractal set defined on the unit sphere. An *arbitrarily small* perturbation to the potential representing the scatterer, completely modifies this set, including its coarse grained properties. But since an arbitrarily small perturbation always exists, the modeling of the scattering experiment using classical point dynamics is an insufficient abstraction. Any meaningful modeling of a physical experiment must incorporate the perturbing effect of the ‘rest of the universe’ in such a way that it can be neglected below a certain threshold. Classical point dynamics — classical electrodynamics to be precise — fails to meet this criterion (and this has nothing to do with chaoticity in the usual sense of exponential sensitivity to initial conditions, but rather with the infinite time a particle gets trapped in chaotic targets).

The above situation drastically changes when modeling the experiment using quantum mechanics. The quantum mechanical differential cross section is always a smooth function, converging to a smooth distribution on the unit sphere as any perturbation to the potential representing the scatterer, or any coupling to the environment, are removed. While practically, it may not always be a problem-free tool for predicting the cross section (when the wavelength of the particle is much smaller than the scale of the potential) the above consistency criterion is always met.

To each scattering event in the crystals there corresponds a radiative perturbation to the aether, containing both advanced and retarded components. These ‘aether waves’ propagate inside and around the interferometer, slightly perturbing the (locally almost) classical path of the cores, in a way which *globally* depends on the configuration of the interferometer. As the dynamics of the cores inside the crystals is chaotic, the aether excitations, their small local effect notwithstanding, have a dramatic effect on the final scattering direction of the particle.

The chaoticity of the underlying classical dynamics is crucial for the operation of the interferometer. Suppose we remove BS2 from the apparatus (b). The influence of the aether excitations on the dynamics of a particle passing in region R is now negligible, and the particle continues its straight classical path, almost unperturbed, as follows from momentum conservation. This should be contrasted with (c), ‘surrealistic’ trajectories predicted by Bohmian mechanics, taking the other direction [2].

We have focused our discussion on a crystal BS as the arena for this chaotic dynamics but, in fact, it is not chaoticity itself — a classical notion — which is essential for the operation of the interferometer, but rather the sensitivity of chaotic dynamics to perturbations. All interferometers, whether electronic or atomic, use BS’s in the form of highly sensitive devices (usually involving the spin of the particle) facilitating the amplification of the small aether induced perturbations to the core’s dynamics.

3.1.5 The ensemble current

The above description of interferometers invites a troubling question. As is well known, interferometers can be tuned to produce nearly deterministic results, with one detector firing some 99% of the times and the other only 1%. If the local dynamics of the particles are so nearly classical, viz. locally defined, then how do they acquire this destiny, of arriving predominantly at one detector rather than the other? (or exit the crystals at the Bragg angles only?)

To answer this question, we first need to see what a scattering experiment is, in the context of ECD. Let $j \in \mathcal{E}$ be the regular electric current associated with a solution realized in the experiment (we drop the r superscript in this section), and \mathcal{E} the ensemble of all such currents. An experiment is seen as a realization of a measure $d\mu(j)$ defined on \mathcal{E} , namely, we assume that as the number, n , of scattered particles goes to infinity, the number of solutions realized in any subset $\Sigma \in \mathcal{E}$ approaches $n\mu(\Sigma) \equiv n \int_{\Sigma} d\mu(j)$. The reader can verify that the scattering cross section as well as any other measurable statistical expression produced by single-body QM, such as the spectrum of atoms, can be read from an ordinary, conserved,

four-current — the *ensemble current* ⁶,

$$j_{\text{ens}} = \int_{\mathcal{E}} d\mu(j) j. \quad (29)$$

Stated in the above terminology, then, the question of interest is why does this current have such an asymmetric form? The answer to the question does not lie in four-dimensional Minkowski's space-time, on which j_{ens} is defined, but rather in infinite-dimensional \mathcal{E} , the domain of μ . A single 'point' in \mathcal{E} — a current j — is such a complex, non locally defined object, that we lack any intuition regarding sensible distributions thereof. Why is 99% — 1% less intuitive than 50% — 50%? Likewise, why is the nonuniform shape of the Hydrogen-atom spectrum counter intuitive? Note that in both cases, no classical counter proposal even exists. In the scattering case, the standard procedure of averaging over the impact parameter leads to a meaningless result when applied to chaotic systems. As to the spectrum — a classical Hydrogen atom is a meaningless concept to begin with.

In fact, the measure μ should be regarded as an *independent law of nature*, on equal footing with ECD itself, constrained only by compatibility requirements with ECD and the experimental settings (try thinking what would constitute a natural μ ?). For this reason, QM enjoys a similar status of an independent law.

3.1.6 Relativistic wave equations

Single-particle QM, as argued above, describes very coarse aspects of the measure μ — very 'low order moments' of that infinite dimensional distribution. It should not come as too great a surprise that, assuming ECD is indeed the physics prevailing at the atomic scale, QM could have been anticipated independently of ECD, with the latter's very unique content. We shall next show why relativistic wave equations, such as the first or second order Dirac equation, and the Klein-Gordon equation, are a natural tool for guessing those moments, in certain cases of a single particle moving in an external field.

⁶The differential scattering cross-section to a given solid angle $d\Omega$ around Ω , for example, is easily deducible from the ensemble current (29). It is just

$$\frac{1}{Q d\Omega} \lim_{x^0 \rightarrow \infty} \int_C d^3 \mathbf{x} j_{\text{ens}}^0, \quad (28)$$

with $Q = \int d^3 \mathbf{x} j^0$ the conserved charge of the particle, and $C = C(d\Omega, \Omega)$ the cone in three space defined by $d\Omega$ and Ω . This follows upon inserting expression (29) into (28). In the limit $x^0 \rightarrow \infty$, every $j^0(x^0, \mathbf{x})$ is entirely supported in C , or in its complement. The \mathbf{x} integration then extracts $Q \chi_{\Sigma}(j)$ with $\Sigma = \Sigma(C) \in \mathcal{E}$ the subset of solutions scattering to cone C , and $\chi_{\Sigma}(\cdot)$ its characteristic function. The result is therefore $(d\Omega)^{-1} \int_{\mathcal{E}} d\mu(j) \chi_{\Sigma}(j) \equiv (d\Omega)^{-1} \mu(\Sigma)$ which is the definition of the differential cross-section.

As yet another example, consider the EM spectrum emitted by a heated gas. For a sufficiently dilute gas, the currents associated with the bound electrons (those generating the radiation) can safely be assumed to constitute an incoherent ensemble \mathcal{E} . By the linearity of Maxwell's equation, and the incoherence assumption, the spectrum produced by the ensemble current equals the sum of spectra produced by the individual currents in \mathcal{E} . Equivalently, \mathcal{E} can comprise different, sufficiently remote time segments, of the current associated with a single atom. The spectral peaks, then, appear simply as dominant frequencies in the dipole radiation of j_{ens} , representing *statistically* more common frequencies in the dipole radiation of members in the ensemble.

Consider, then, the ECD solution of a single particle in the external field F_{ext} . Let this solution be indexed by the electric current j , associated with the particle, and let ${}^j m$ be the corresponding e-m tensor. From (19) we have

$$\partial_\nu {}^j m^{\nu\mu} = (F_{\text{ext}}^{\mu\nu} + {}^j F_{\text{sel}}^{\mu\nu}) j_\nu, \quad (30)$$

with ${}^j F_{\text{sel}}$ the self-field generated by j ,

$$\partial_\nu {}^j F_{\text{sel}}^{\nu\mu} = j^\mu. \quad (31)$$

Multiplying (30) by $d\mu(j)$ and integrating over \mathcal{E} , we first make the assumption that the contribution of the self-fields,

$$\int_{\mathcal{E}} d\mu(j) {}^j F_{\text{sel}}^{\mu\nu}(x) j_\nu(x), \quad (32)$$

can be neglected, compared with that of the external field. This is a reasonable assumption for a sufficiently incoherent ensemble, as then the self contribution of different members in the ensemble to the self force at a point x , enters with a nearly random orientation. For this to happen, however, the different charges must not radiate (advanced or retarded fields) in preferred directions, hence the limitation of the ensemble current approach and of relativistic wave equations in particular (see more in section 3.2.1). Note, nonetheless, that the self-fields are dominant in the individual currents j and ${}^j m$, even when their contributions to the integral over the ensemble have been neglected, guaranteeing that self-force effects are not eliminated in that process. In particular, the effective mass and charge of the particles, strongly depend on that self field.

With the above approximation, we get the following four relations

$$F_{\text{ext}}^\mu j_{\text{ens}}^\nu = \partial_\nu m_{\text{ens}}^{\nu\mu}, \quad \text{with} \quad m_{\text{ens}} = \int_{\mathcal{E}} d\mu(j) {}^j m, \quad (33)$$

and a conservation constraint

$$\partial_\nu j_{\text{ens}}^\nu = 0, \quad (34)$$

inherited from the conservation of the individual j . The Lorentz vector and second rank tensor, j_{ens} and m_{ens} resp., must obviously transform like their constituents in any symmetry transformation belonging to the symmetry group of ECD.

Consider now a low energy scattering experiment. As shown above, the scattering cross section can be computed from j_{ens} . However, a similar construction applied to m_{ens} can also produce the cross section, which *must coincide* with that computed using j_{ens} . This relation adds up to (33), (34) and the symmetry group of ECD, producing a very restrictive condition on the set of permissible pairs $\{j_{\text{ens}}, m_{\text{ens}}\}$, regardless of the details of the ECD dynamics.

A systematic way of producing such constrained pairs, enjoying the full symmetry group

of ECD, is via relativistic wave equations⁷. In the scalar case, the relevant equation is the Klein-Gordon equation

$$(D^2 + \hat{m}^2) \psi = 0, \quad (37)$$

with the gauge covariant derivative

$$D = \hat{h}\partial - i\hat{q}A, \quad (38)$$

where \hat{h} , \hat{q} and \hat{m} are some constants, and A the external EM potential. The expressions for the ensemble electric current

$$j_{\text{ens}}^\mu = \hat{q} \text{Im } \psi^* D^\mu \psi, \quad (39)$$

and the ensemble e-m tensor

$$m_{\text{ens}}^{\nu\mu} = g^{\nu\mu} \left(\frac{1}{2} \hat{m}^2 \psi \psi^* - \frac{1}{2} (D^\lambda \psi)^* D_\lambda \psi \right) + \frac{1}{2} (D^\nu \psi (D^\mu \psi)^* + \text{c.c.}), \quad (40)$$

satisfy all the above compatibility conditions — eq. (33) in particular. Eq. (33), when restricted to a field-free region, imposes certain relations between the parameters of (37), and the conserved electric charge and mass of the particles comprising the ensemble.

The wave-function ψ , then, labels an ‘irreducible ensembles’, μ_ψ , to which there corresponds an ‘irreducible pair’, $\{j_{\text{ens}}, m_{\text{ens}}\}_\psi$. A generic experiment, however, involves a few irreducible ensembles, which are sampled with different weights. This is the meaning of a ‘statistical mixture’ of wave-functions in QM. The collapse postulate of measurement theory, merely represents a transition to a sub-ensemble.

A major historical difficulty associated with the KG field is also resolved in this framework. The non-positivity of j_{ens}^0 (motivating the Dirac equation) simply reflects the non-positivity of the individual j^0 comprising j_{ens} . It is only the space integral over those individual components, representing the total charge, that is guaranteed to remain constant. Finally, one can now understand the accuracy of single particle QM, despite ignoring self interaction effects.

⁷The more general way is via the conservation laws associated with solutions, ψ , of the five dimensional Schrödinger equation (15). Its unitarity implies

$$\partial_s |\psi|^2 = \partial \cdot J \equiv \partial \cdot (\text{Im } \psi^* D \psi), \quad (35)$$

while the Ehrenfest relations give

$$\begin{aligned} \partial_s J^\mu &= F^{\mu\nu} J_\nu - \partial_\nu M^{\nu\mu} \\ &\equiv F^{\mu\nu} J_\nu - \partial^\nu \left[g^{\nu\mu} \left(\frac{i\hbar}{2} (\psi^* \partial_s \psi - \partial_s \psi^* \psi) - \frac{1}{2} (D^\lambda \psi)^* D_\lambda \psi \right) + \frac{1}{2} (D^\nu \psi (D^\mu \psi)^* + \text{c.c.}) \right] \end{aligned} \quad (36)$$

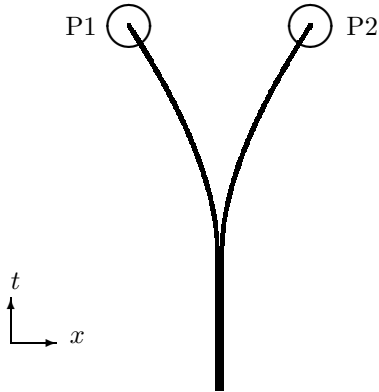
Integrating (35) and (36) from $s = -\infty$ to $s = \infty$, we get two candidates, $j_{\text{ens}} = \int ds J$ and $m_{\text{ens}} = \int ds M$, satisfying all our requirements. These, however, correspond to ensembles with a continuum of masses, and are therefore more difficult to relate to actual experiments, involving a single particle species.

3.2 Many-body ECD

In the previous section, the EM potential was divided into an external potential, generated by all particles but one, plus a self potential, due entirely to this one, privileged, particle. This division is legitimate on the premise that the self potential of the privileged particle does not alter the solutions of the rest of the particles, which is not the case when the privileged particle interacts with the rest of the particles, either ‘electrostatically’, viz. at close range, or ‘radiatively’, via long-range aether waves.

Let us begin with the first case. As explained in section 2.3, the self consistent potential entangles closely interacting particles in such a way that one can no longer regard matter as a composition of individual particles but, instead, as some ‘self consistent matter-radiation condensate’. What may seem surprising at first is that long after their separation, and at arbitrarily remote locations, two particles which have closely interacted in the past, ‘bear the memory’ of their encounter.

Consider, for example, two nucleons, escaping a nucleus, arriving each at a polarimeter (P1 and P2).



If the two polarimeters are positioned sufficiently far apart, then the ECD system, self potential included, can be solved independently for each particle. This is a consequence of $G(x, x', s) \xrightarrow{s \rightarrow 0} 0$ sufficiently fast, suppressing the large $|s - s'|$ contribution to the s' -integral in (13). There is therefore nothing unique about the dynamics of each particle, giving away their histories, which is washed away over macroscopic scales. The above independence notwithstanding, if one tries to continue those independent solutions into the past, then at some point, when the two particles come close, it could become impossible to combine the two solutions into a single, self consistent one, unless each of the two independent solutions is restricted to a certain subset of the full set of independent solutions — a subset which obviously depends on the orientation of *both* polarimeters. Given the orientations of the two polarimeters, therefore, the combined solution of the two particles in the above example, must be solved as a whole — as a single space-time structure.

To each orientation choices for P1 and P2, there corresponds a *different* ensemble, \mathcal{E} , of two-particle ECD solutions, equipped with its own measure, μ . This contradicts Bell’s assumption in deriving his celebrated inequalities, which maintains that the *same* ensemble

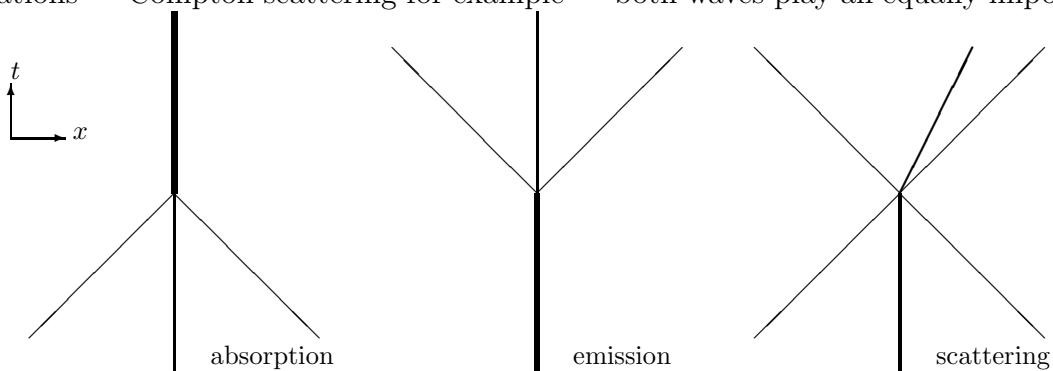
of hidden variables be used, irrespective of the orientations of the polarimeters (roughly corresponding to the fact that the particles should not ‘anticipate’ the orientations of the polarimeters before encountering them).⁸

Like its single-body counterpart, the measure, μ , enjoys the status of an independent law of nature, on equal footing with ECD itself. However, a simple generalization of the ensemble current to the case of many-body ECD, probably doesn’t exist, hence the enormous complication of many-body relativistic QM — quantum field theory — involving both matter and the EM potential.

Finally, let us note that the same discussion holds also for two particles, initially separated, which later bind together. This scenario, however, does not correspond to common experiments.

3.2.1 The conspiracy leading to the invention of the photon

Perhaps the strongest motivation for the introduction of photons, is the salvation of energy-momentum conservation. Indeed, the photoelectric and Compton’s effects are manifestly in violation of classical energy-momentum conservation. More specifically, equation (24), expressing the change in the four-momentum of a particle as a function of the integrated e-m flux across a space-like surface surrounding the particle, can formally be applied to the corresponding classical currents as well. In the case of the photoelectric or Compton’s effects, the e-m flux across T , identical with the Poynting vector, is computed from the external trigger, F_{ext} , and a possible retarded outgoing wave, generated in the jolting of the charge. As both effects are observed even for extremely feeble triggers, the contribution of F_{ext} to the Poynting vector may be neglected, while that of the retarded wave can be shown to be positive. We may then get an arbitrarily large excess of energy at times following the jolting of the charge. In ECD, on the other hand, we saw that advanced e-m waves must be included in the analysis. Thus, for example, ‘photon absorption’ by a molecule, should correspond to predominantly incoming advanced e-m waves, converging on the molecule and increasing its internal energy (or ionizing it as in the photoelectric effect). In ‘spontaneous emission’, it is rather outgoing retarded waves removing energy from the molecule. In other situations — Compton scattering for example — both waves play an equally important role.



⁸The above mechanism, accounting for violations of Bell’s inequalities, is in the spirit of so called ‘retro-causal’ models. See, e.g. [1] and extensive references therein.

The above two features, viz. automatic selection of the correct radiation field (guaranteeing energy-momentum conservation), along with its incorporation into the dynamics of the particle (no self-force problem), are missing from classical electrodynamics, hence the need for ECD to implement this, otherwise, classical idea.

Being highly nonlocal and nonlinear, there are plenty of pairs $\{\phi, \gamma\}$, solving the ECD system (self potential included) for a given external trigger. The distribution of the corresponding currents can be read from the appropriate ensemble-current (see section 3.1.5) which, in the case of the photoelectric effect, gives the well known⁹ result that the electron either jolts with energy, $\lambda\omega$, proportional to the frequency, ω , of the incident radiation (ignoring for simplicity the binding energy), or else does not jolt at all. This binary response of electrons, typical of *all* ‘*photodetectors*’ *by definition* (or else they are called calorimeters, antennas, etc.), is the historical reason for the introduction of photons. It is as if a ‘light corpuscle’ of energy $\lambda\omega$ has struck the jolted electron.

Yet another standard result emerging from the analysis of the ensemble current which contributes to the illusion of a photon, is that the probability for a jolting event is proportional to the amplitude squared of the incident wave, implying that the probability drops as the inverse of the distance squared between source and detector — just as if a flux of particles is erupting from the emitter.

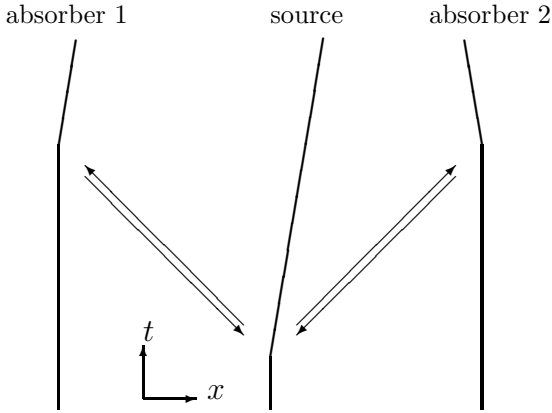
But the analogy with other particles goes even further. A typical example involves a so called ‘single-photon source’ or more generally an n -photon source (Fock state source), e.g. a molecule excited by a femtoseconds laser pulse, and then allowed to spontaneously decay. If the source is surrounded by a large sphere, consisting of independently operating photodetectors (which can further be prevented from cross-talking by, e.g. partitions) then the above results of the ensemble current, imply that the *average* number of photodetections does not depend on the radius of the sphere, and is entirely an attribute of the source (note that, as the expectation value is additive even for dependent random variables, this result is not altered when, latter, we argue that the photodetectors are not independently operating). This, again, is consistent with a scenario of a release of a fixed number of particles in each decay of the molecule. However, the independence of the different photodetectors also imply that the number of detected photons should fluctuate around its mean with a standard deviation proportional to the square-root of the mean. For a large mean, this fluctuation may be ignored, but for a small one, it is significantly greater than the observed value which is more consistent with a fixed number of particles scenario, having no fluctuations.

As implied above, the loop-hole in the analysis is in the assumption of independence of the

⁹This calculation is usually performed with the non-relativistic Schrödinger equation, considering the incident wave as a small perturbation. However, for wavelength much smaller than the electron’s Compton length, the Dirac equation gives identical results.

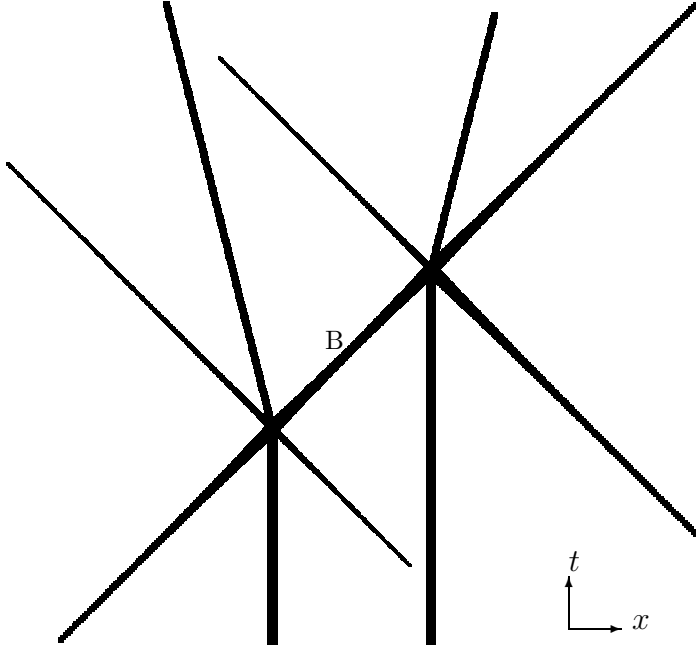
The success of the ensemble current formalism in the case of the photoelectric effect, is due to the isotropic distribution of the direction of the ejected particle, hence also of the corresponding self force, justifying the omission of the term (32). In contrast, this formalism fails when applied to Compton scattering, i.e. an external EM trigger in the form a plane wave, but without a heavy trap holding the particle. Momentum conservation — ignored in the photoelectric effect due to the large mass of the trap — dictates that the direction of the ejected charges, must be strongly correlated with that of the incident wave, and (32) cannot be neglected.

photodetections. While it is possible to prevent different photodetectors from cross-talking, it is, by definition, impossible to prevent each of them from cross talking with the source if advanced waves are present in the radiation fields of the absorbing charges¹⁰. In actual experiments, e.g. [6], the retarded field of the source is relayed to the detecting charges by other charges, comprising mirrors, beam-splitters, fiber-optics etc. The crucial point is that, whatever optical path exists between the source and the detector, by means of retarded fields, there must necessarily exist a reverse path leading from the detector to the source via advanced fields. The source therefore serves as a hub for indirect cross-talking between the absorbing charges, leading to statistical dependence in their responses. As to why the actual fluctuation around the mean is much smaller, rather than larger, than that expected on the premise of independence — this is a statistical issue, not to be sought in ECD alone. This is the realm of QM — QED to be specific. Violations of Bell’s inequalities in photons pair measurements etc., are presumably all manifestations of that indirect cross-talking.



We see how various features of ECD and QM may have conspired to bring about the illusion that ‘light particles’ must be involved in radiation processes (one can potentially extend the above arguments to other neutral particles, such as the neutrino). The real moral, however, lies in the geometry of Minkowski’s space and in the unity of the e-m field p . We have previously argued that the self consistency loop entangles, in the statistical sense, two particles whose associated world-cylinders, supporting the lion’s share of their charges, have a significant overlap in \mathbb{M} . This can be seen as a manifestation of the fact that, fundamentally, the value of p at a point cannot be attributed to any single particle, not even inside the dense cylinders associated with the particle. The conclusion deduced from that example generalizes to the observation that any connected volume in \mathbb{M} , of a sufficiently high e-m density must be treated as a single space-time structure. In particular, the following densely charged connected structure, is typical of all emitter-absorber ‘transactions’.

¹⁰The use of advanced solutions in order to explain the non classical statistics of photons, latter receiving the name ‘the transactional interpretation of QM’, is described in [3]. Using point charges, however, that proposal does not explicitly deal with energy-momentum balance, nor with the mechanism causing a charge to jolt.

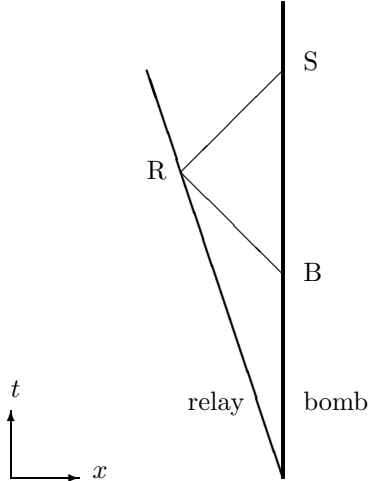


The term ‘transaction’ is deliberately borrowed from [3] as it highlights the symmetric role played by both charges appearing in the structure, viz. the absorber may just as well be seen as the cause, triggering the emission via advanced waves, rather than the effect, triggered by the retarded waves of the emitter. Note that in ECD this blurring between cause and effect goes even further than in [3], as p , in particular on the ‘bridge’, B, between the two particles, cannot be decomposed into advanced plus retarded contributions, and is therefore a genuine attribute of the structure as a whole.

3.2.2 Advanced waves

The central role played by advanced waves in explaining the illusion of a photon, calls for a closer look at these disputable objects. There is a strong, largely unjustified bias against the inclusion of advanced waves — advanced solutions of Maxwell’s equations in particular — into the description of physical reality. The main objection draws parallels with ‘contrived’ advanced solutions of other physical wave equations (e.g. surface waves in a pond converging on a point and ejecting a pebble.). This parallelism, however, is a blatant repetition of the historical mistake, which led to the invention of the aether (the historical aether, not to be confused with that used in this paper). The formal mathematical similarity between the d’Alembertian (the only linear, Lorentz invariant second-order differential operator) and other (suitably scaled) wave operators, is no more than a mis-fortunate coincident (has this coincidence had some real substance to it, then application of the Lorentz transformation to the wave equation describing the propagation of sound, for example, would have yielded a meaningful result).

Yet another argument against advanced solutions is their alleged involvement in causal paradoxes.



Indeed, if advanced waves could be generated just like their retarded counterparts, then the following paradoxical situation could occur. A device consisting of a bomb, a transmitter, a receiver and a timer, is set to send a retarded signal at S. The signal is relayed at R, received at B, and triggers the fuse of the bomb. But if the bomb goes off at B, then no signal is sent at S. Why then did the bomb explode? On the other hand, if the bomb does not go off at B, then a signal must be sent at S, detonating the bomb at B. Either way get a contradiction.

The resolution of the paradox should not be sought in ECD proper. Indeed, if ECD is a valid theory, then the CPT image of a radio transmitter sending retarded waves, is a radio transmitter made of antimatter sending advanced waves. A radio transmitter, however, cannot be seen as an autonomous entity. Its generated waves are eventually absorbed by other particles and, as argued above, the emission of waves cannot be separated from their absorption (as in the Wheeler-Feynman absorber theory, [7]). The privileged status of retarded waves in all *macroscopic* radiation processes, is therefore an attribute of the specific *solution* of the ECD equations (selected, among else, by the anthropic principal), representing the local part of the universe we live in, and is intimately connected with the excess of matter over antimatter around us.

It seems, then, that the strongest case against advanced solutions is observational. While spontaneous emission or absorption may be seen as direct evidences to the contrary, if advanced solutions played a dominant role in *any* photo-absorption (as implied e.g. in the figure on page 21) then their prevalence should have matched that of their retarded counterparts, leaving a striking signature on all radiation processes. Let us then see why this needs not be the case. Recall that our primary motivation for introducing advanced solutions was to salvage energy-momentum conservation. In the example from the previous section of a source surrounded by photodetectors, the integrated flux of energy falling on any single photodetector, must be smaller than the EM energy released by the source. If that energy equals $\lambda\omega$, advanced waves must be invoked in order to account for the firing of a photodetector, as this amounts to increasing the energy of an electron by $\lambda\omega$. However, if the above ‘single photon source’ is replaced by a continuous light source of arbitrary intensity, then the following pro-

cess, not involving advanced waves, can be envisaged. Retarded waves, originating from the source, arrive at the photodetector, which slowly absorbs them. In the process of absorption the electrons in the device radiate in the same direction as that of the incident radiation, but in a phase which interferes destructively with the latter, slowly ‘removing’ energy from the incident wave. This is essentially the classical description of radiation absorption, only in ECD, the energy extracted from the incident wave needs not appear instantly as kinetic energy. The extended support of the ECD energy density, in conjunction with its ability to evolve over time, support a scenario in which energy is gradually accumulated by the charge in the form of latent ‘internal’ energy, and is rapidly converted into kinetic energy only when a threshold, equal to $\lambda\omega$, has been crossed. That the conversion of latent energy into kinetic energy happens at the $\lambda\omega$ threshold can, again, be read from the ensemble current which, as remarked before, is indifferent to the mechanism shooting the individual electrons.

Remarkably, it is known that the statistics of photodetection also changes when shifting to a continuous source. When the readings of two photodetectors are correlated (as in [6]), the anticorrelation consistent with a particle scenario, turns into the expected positive correlation when the single photon source is replaced by a continuous light source of thermal origin, or to (the equally intuitive) vanishing correlation when strongly attenuated laser light is used. It appears, therefore, that advanced waves play a dominant role only in sufficiently ‘delicate’ radiation process, involving energy transfer on the order of $\lambda\omega$. Such processes are overwhelmed by ordinary radiation processes involving a huge number of particles, such as the burning of a candle, or in lasing devices. Finally, here is a direct prediction of ECD: If the the photodetectors in [6] are replaced by delicate thermometers, the anticorrelation observed for ‘single photon sources’ will turn into a correlation in the readings of the temperature, proving that photons are merely statistical artifacts of certain detection devices.

4 discussion

At the heart of any physical theory is a labeling scheme for events in space-time, viz., a coordinate system. Much of the development of theoretical physics over the years can be seen as a gradual increase in the flexibility of choosing a coordinate system for space-time, naturally accompanied by increasingly severer constraints on candidate physical theories, consistent with that greater flexibility. The freedom of choosing an arbitrary scale for a coordinate system, endorsed in the current paper, increases the (already large) set of permissible coordinate systems related to each other by a Poincaré transformation, but at the same time necessitated a very unusual mathematical construction in order for ECD to comply with scale covariance. The ultimate step in that direction is, of course, general covariance — the freedom to choose an arbitrary coordinate system to label space-time. This is not only an esthetically appealing symmetry, but it also avoids the circularity involved in the definition of inertial frames (to corroborate the laws of physics defined in inertial frames, one first needs to construct an inertial frame, which is only defined by the condition that the laws of physics hold in it). This ultimate freedom, nonetheless, does not come without a heavy price. At least one tensorial field — a ‘metric’ — must be involved (there are no generally

covariant theories of lower rank tensors), the affine structure of Minkowski's space is lost, and only generally covariant quantities are meaningful.

Matter-free general relativity is essentially the simplest generally covariant theory. However, trying to add matter to it in the simple form of point particles, creates a self-force problem similar to the one plaguing classical electrodynamics of point charges. The metric becomes ill defined exactly on the world-lines of the particles and consequently, general relativity — strictly speaking — is not even a theory. Using (by now) standard techniques, one can easily generalize ECD to curved space-time, offering at once a generally covariant (scale covariance being a sub group of the former) resolution of the gravitational self-force problem (along with other singularity problems originating from the use of point-sources, such as a black hole singularity).

Space-time structures in generally covariant ECD are described by a covariantly conserved symmetric matter e-m tensor (or stress-energy tensor), $\nabla_\nu p^{\nu\mu} = 0$, depending also on the metric but not containing 'pure gravitational e-m' (which cannot be covariantly defined in GR; the covariant conservation of p , rather than the naive $\partial_\nu p^{\nu\mu} = 0$ conservation, incorporates gravitational effects). The central ECD system does not change its form and still guarantees that no electric charge, nor e-m leak into world sinks associated with individual particles.

Conceptually, however, generally covariant ECD adds nothing new to flat space ECD. The 'scaffold' used to construct space-time structures is of course different, as well as the latter's physical attributes (only generally covariant ones), but there is still a single measurable entity — the aether — focused around world lines which we call particles, and filling the entire space in between those world lines. This interparticle aether, we argued in section 3.1.3 in the context of scattering experiments, is the cause of the small deflections of particles, viz., their cores, from classical paths. Those tiny deflections give rise to observable effects only in sufficiently isolated systems, free of direct forces which tend to have a variability greatly exceeding the small influence of the aether. Intergalactic space, as well as intragalactic regions sufficiently far from a galaxy's core, are places in which Einsteinian gravity is extremely feeble, allowing for the influence of the aether to leave an observational mark — possibly a striking one (note that the presence of a massive body, such as a galactic core, influences the surrounding aether also at great distances). The possible implications to the dark matter problem, and to the MOND phenomenology in particular, are obvious.

Yet another feature of (generally covariant) ECD which may be relevant to astronomy and cosmology is entanglement. By the principle of scale covariance, the descriptive jurisdiction of ECD must not be limited to scales, we human consider small, and the morphology of galaxies, or even the universe as a whole, may find their explanation in the entanglement mechanism of ECD, either at the single structure level or in the corresponding (yet undeveloped) statistical theory. Indeed, as entanglement is transitive, all the particles in a dense cloud are entangled, and should be solved as a single space time structure. If a large scale astronomical object, such as a galaxy, passes through an epoch of a dense 'fireball' (not necessarily anything as dramatic as a big bang or big crunch — whatever that means) it could reasonably be that its morphology and dynamics at much later/earlier stages, reflect that entangled epoch. Put

differently, the morphology of the galaxy in three space, is but a space-like cross section of the full space-time structure, and it is only in the latter — past and future parts — that the former can be understood (much like violations of Bell’s inequalities). Gravitation on galactic scales (and possibly above) could therefore be due to such a primordial entanglement, and need not even resemble solar scale gravity (the largest scale in which general relativity has been directly confirmed). This would have obvious implications on the current interpretation of astronomical data. In this regard, we should also mention another possibility opened by ECD — scale drift. As shown in appendix C, both the mass of individual ECD particles, as well as the scale charge of their combined solution, may slowly drift over time. This offers an alternative explanation for the source of galactic redshifts. In fact, a universe collectively increasing its scale leads to a Hubble-like relation, as light collected from remote galaxies is emitted at an epoch of lower mass (hence longer wavelength) which is proportional to the distance between the emitter and the observer, for *all* observers.

Finally, as we saw already in the context of simple scattering experiments, ECD — the rule governing the shape of individual space-time structures — is far from being a complete description of reality. There must be another law of nature, dealing with statistical aspects of ensembles of structures. We saw that QM addresses this question in some very specific situations, e.g. a scattering experiment, but by no means in all situations. To answer all conceivable statistical questions, one needs a full knowledge of the relevant ensemble, which is beyond the reach of current quantum theory, describing some very ‘low moments’ of the full distribution/measure characterizing the ensemble. It is therefore only natural to seek extensions of quantum mechanics, dealing with more general statistics of ensembles or even with different ensembles altogether — ensembles of chaotic systems, in particular. Just like the example of the interferometer showed, partial to complete predictability may emerge from such a statistical description of an ensemble of chaotic systems. The mysterious procedure of ‘quantizing’ a dynamical system, may very well be just the right way of associating a statistical theory with a single-system theory, and could be applicable to macroscopic systems as well, potentially providing a novel tool for the prediction of the long-term behavior of chaotic systems.

Appendices

A The ‘refined’ central ECD system and regularized currents

Both equations, (13) and (14), comprising the central ECD system, involve a delicate $\epsilon \rightarrow 0$ limit, requiring clarifications. Focusing first on (13), we see that, for fixed γ and G , it is in fact an equation for a function $f^R(s) \equiv \phi(\gamma_s, s)$. Indeed, plugging an ansatz for f^R into the r.h.s. of (13), one can compute $\phi(x, s) \forall s, x$, and in particular for $x = \gamma_s$, which we call $f^L(s)$. The linear map $f^R \mapsto f^L$ (which, using $G(x', x; s) = G^*(x, x'; -s)$, can be shown to be formally self-adjoint) must therefore send f^R to itself, for (13) to have a solution. Now, the universal, viz. A -independent, $i/(2\pi\hbar s)^2$ divergence of $G(y, y, s)$ for $s \rightarrow 0$, implies

$f^{\text{R}} \mapsto f^{\text{R}} + O(\epsilon)$, so the nontrivial content of (13) is in this $O(\epsilon)$ term, which we write as ϵf^{r} (‘r’ for residue), with $f^{\text{r}} = O(1)$ for $\epsilon \rightarrow 0$. In [4], $\lim_{\epsilon \rightarrow 0} f^{\text{r}} = 0$ was implied as the content of (13). While this may turn out to be true for some specific solutions (a freely moving particle, for example), the Equation (13) then takes the equation should take a more relaxed form

$$\text{Im} \left(\lim_{\epsilon \rightarrow 0} f^{\text{r}*} \right) f^{\text{R}} = 0, \quad (41)$$

where, as usual, ‘Im’ is the imaginary part of the entire product to its right.

Moving next to the second ECD equation, (14), conveniently rewritten as

$$\text{Re} \bar{h} \partial_x \phi(\gamma_s, s) \phi^*(\gamma_s, s) = 0, \quad (42)$$

a similar isolation of the nontrivial content exists. For further use, however, we first want to isolate the contribution of the small s divergence of G to $\phi(x, s)$, for a general x other than γ_s . To this end, we need the small- s form of the propagator G . Plugging the ansatz

$$G(x, x', s) = G_{\text{f}} e^{i\Phi(x, x', s)/\bar{h}} \quad (43)$$

into (15), with

$$G_{\text{f}}(x, x'; s) = \frac{i}{(2\pi\bar{h})^2} \frac{e^{\frac{i(x-x')^2}{2hs}}}{s^2} \text{sign}(s), \quad (44)$$

the free propagator computed for $A \equiv 0$, and expanding Φ (not necessarily real) in powers of s , $\Phi(x, x', s) = \Phi_0(x, x') + \Phi_1(x, x')s + \dots$, higher orders of Φ_k can recursively be computed with Φ_0 alone incorporating the initial condition (16) in the form $\Phi_0(x', x') = 0$ (note the manifest gauge covariance of this scheme to any order k). For our purpose, Φ_0 is enough. A simple calculation gives the gauge covariant phase

$$\Phi_0(x, x') = q \int_{x'}^x d\xi \cdot A(\xi), \quad (45)$$

where the integral is taken along the straight path connecting x' with x . Substituting (43) into (13), and expanding the integrand around s to first order in $s' - s$: $\gamma_{s'} \sim \gamma_s + \dot{\gamma}_s(s' - s)$, $\Phi_0(x, \gamma_{s'}) \sim \Phi_0(x, \gamma_s)$, $\phi(\gamma_{s'}, s') \sim f^{\text{R}}(s)$, leads to a gauge covariant definition of the *singular part of ϕ*

$$\phi^{\text{s}}(x, s) = f^{\text{R}}(s) e^{i(\Phi_0(x, \gamma_s) + \dot{\gamma}_s \cdot \xi)/\bar{h}} \text{sinc} \left(\frac{\xi^2}{2\bar{h}\epsilon} \right) \quad (46)$$

with $\xi \equiv x - \gamma_s$. Consequently, the *residual* (or *regular*) wave-function is defined via the gauge covariant equation

$$\epsilon \phi^{\text{r}}(x, s) = \phi(x, s) - \phi^{\text{s}}(x, s). \quad (47)$$

Using $\partial_x \Phi_0(x, \gamma_s)|_{x=\gamma_s} = A(\gamma_s)$, we have

$$\phi^{\text{s}}(\gamma_s, s) = f^{\text{R}}(s), \quad \bar{h} \partial_x \phi^{\text{s}}(\gamma_s, s) = i[\dot{\gamma}_s + A(\gamma_s)] f^{\text{R}}(s), \quad (48)$$

and (42) is automatically satisfied up to an $O(\epsilon)$, gauge invariant term

$$\epsilon \operatorname{Re} \bar{h} \partial_x [\phi^r(\gamma_s, s) \phi^s(\gamma_s, s)^*] = \epsilon \operatorname{Re} D\phi^r(\gamma_s, s) \phi^s(\gamma_s, s)^*, \quad (49)$$

where the above equality follows from (48), $\phi^r(\gamma_s, s) = f^r(s)$ and (41). The refined definition of (14) is therefore

$$\lim_{\epsilon \rightarrow 0} \operatorname{Re} D\phi^r(\gamma_s, s) \phi^s(\gamma_s, s)^* = 0. \quad (50)$$

Using the above definitions, (41) can also be written as

$$\lim_{\epsilon \rightarrow 0} \operatorname{Im} \phi^r(\gamma_s, s) \phi^s(\gamma_s, s)^* = 0. \quad (51)$$

More insight into this refinement of the central ECD system is given in the sequel. For the time being, let us just note that it is invariant under the original symmetry group of ECD. In particular, the system is invariant under

$$\phi^s \mapsto C \phi^s, \quad \phi^r \mapsto C \phi^r, \quad C \in \mathbb{C}, \quad (52)$$

under a gauge transformation

$$A \mapsto A + \partial \Lambda, \quad G(x, x', s) \mapsto G e^{i[q\Lambda(x) - q\Lambda(x')]/\hbar}, \quad \phi^s \mapsto \phi^s e^{iq\Lambda/\hbar}, \quad \phi^r \mapsto \phi^r e^{iq\Lambda/\hbar}, \quad (53)$$

and under scaling of space-time

$$\begin{aligned} A(x) &\mapsto \lambda^{-1} A(\lambda^{-1} x), \quad \epsilon \mapsto \lambda^2 \epsilon, \quad \gamma(s) \mapsto \lambda \gamma(\lambda^{-2} s), \\ \phi^s(x, s) &\mapsto \lambda^{-2} \phi^s(\lambda^{-1} x, \lambda^{-2} s), \quad \phi^r(x, s) \mapsto \lambda^{-2} \phi^r(\lambda^{-1} x, \lambda^{-2} s), \end{aligned} \quad (54)$$

directly following from the transformation of the propagator under scaling

$$A(x) \mapsto \lambda^{-1} A(\lambda^{-1} x) \Rightarrow G(x, x'; s) \mapsto \lambda^{-4} G(\lambda^{-1} x, \lambda^{-1} x'; \lambda^{-2} s).$$

Regarding this last symmetry, two points should be noted. First, for a finite ϵ it relates between solutions of *different* theories, indexed by different values of ϵ . It is only because ϵ is ultimately eliminated from all results, via an $\epsilon \rightarrow 0$ limit, that scaling can be considered a symmetry of ECD. The second point concerns the scaling dimension, -2 , of ϕ^s and ϕ^r . By the symmetry (52), that dimension can be arbitrarily chosen. However, the central ECD system is but a part of the ECD formalism, which dictates this special choice of dimension to comply with scale covariance (see next section).

A.1 Regularized currents

The $\epsilon \rightarrow 0$ limit of the current (11), as also the limits of all other currents in ECD, vanishes everywhere (except on the world line, $\bar{\gamma} \equiv \cup_s \gamma_s$, traced by γ , where it is finite), trivializing ECD. To correct this situation, two steps are taken. Utilizing the symmetry (52), we first rescale $\phi \mapsto \epsilon^{-1} \phi$, granting j in (11) a nonsingular support. As we show next, however,

the resultant current diverges everywhere in the limit $\epsilon \rightarrow 0$. To fix this new problem, we substitute $\phi \mapsto \epsilon^{-1}\phi^s + \phi^r$ in (11), and note that this divergence, as well as those of all other ECD currents, can be traced to gauge invariant contributions of bilinears in ϕ^s , which in this case read

$$\int_{-\infty}^{\infty} ds q \operatorname{Im} \frac{\phi^{s*}}{\epsilon} D^\mu \frac{\phi^s}{\epsilon} \equiv j^s. \quad (55)$$

Indeed, by (46) we get

$$j^s(x) = \frac{q}{h} \int ds (\dot{\gamma}_s - qA(x) + \partial_x \Phi(x, \gamma_s)) |f^R(s)|^2 \frac{1}{\epsilon^2} \operatorname{sinc}^2 \left(\frac{(x - \gamma_s)^2}{2h\epsilon} \right). \quad (56)$$

Using (in the distributional sense) $\epsilon^{-1} \operatorname{sinc}^2(\epsilon^{-1}y) \rightarrow \pi \delta(y)$ for $\epsilon \rightarrow 0$, we see that j^s contains an ϵ^{-1} term in its ϵ -expansion. Taking further into account the finite width of $\epsilon^{-1} \operatorname{sinc}^2(\epsilon^{-1}y)$ (as oppose to a delta distribution) and its evenness, it can be shown that the next higher power in the expansion is ϵ^1 . This leads to the definition of the *regular current*, j^r — a gauge invariant expression defined as the free coefficient in the ϵ -expansion of j , or equivalently,

$$j^r = \lim_{\epsilon \rightarrow 0} (j - j^s). \quad (57)$$

This regular current is the electric current ultimately associated with an ECD charge, entering as a source into Maxwell's equations (4). By (54), j^r has dimension -3 , consistent with the scaling dimension of A , namely, (4) is invariant under

$$A(x) \mapsto \lambda^{-1} A(\lambda^{-1}x), \quad j^r(x) \mapsto \lambda^{-3} j^r(\lambda^{-1}x), \quad (58)$$

establishing the scale covariance of ECD. Finally, we note that for $A \equiv 0$ and a freely moving γ ($\gamma_s = us$), j^r vanishes, as it must on self consistency grounds.

In appendix C we prove that the regular current, (57), is conserved for $x \notin \bar{\gamma}$. The conservation of a current defined on $\mathbb{M}/\bar{\gamma}$, however, does not imply the time independence of the associated charge $Q = \int d^3\mathbf{x} j^{r0}(x^0, \mathbf{x})$, due to a possible ‘leakage’ of charge into a sink of j^r on $\bar{\gamma}$ or ‘emergence’ of charge from a source thereon. Remarkably, the refined ECD equation (51), turns out to be exactly the condition guaranteeing that no such leakage occurs. Likewise, the second refined ECD equation, (50), guarantees that no energy or momenta leak into sinks of the conserved energy-momentum tensor on $\cup_k {}^k\bar{\gamma}$. It is therefore natural to add to the central ECD system the proviso that the electric charge of each particle, as well as the collective e-m of the system, do not ‘leak to infinity’ (although it is possible that at least the first condition is automatically satisfied).

B The semiclassical approximation of the central ECD system

The central ECD system appears completely detached from its simple classical counterpart (1). Let us then show that they are actually very similar. The semiclassical, or small \hbar

analysis of the central ECD system, is facilitated by the leading order in the \bar{h} expansion of the propagator, G , known as the *semiclassical propagator*¹¹

$$G_{\text{sc}}(x, x'; s) = \frac{i \text{sign}(s)}{(2\pi\bar{h})^2} \sum_{\beta} \mathcal{F}_{\beta}(x, x'; s) e^{iI_{\beta}(x, x'; s)/\bar{h}}. \quad (59)$$

Here, β runs over the different classical paths, viz. paths solving (1) for the fixed A , such that $\beta(0) = x'$ and $\beta(s) = x$; I_{β} is the corresponding action of the path,

$$I_{\beta} = \int_0^s d\sigma \frac{1}{2} \dot{\beta}_{\sigma}^2 + qA(\beta_{\sigma}) \cdot \dot{\beta}_{\sigma}, \quad (60)$$

and \mathcal{F} — the so called Van-Vleck determinant — is the gauge-invariant classical quantity, given by the determinant

$$\mathcal{F}(x, x'; s) = \left| -\partial_{x_{\mu}} \partial_{x'_{\nu}} I_{\beta}(x, x'; s) \right|^{1/2}. \quad (61)$$

Let us next show that to leading order in \bar{h} , the refined central ECD system is solved by any classical γ (in the given EM potential A), and by a corresponding ansatz of the form

$$f^{\text{R}}(s') = C e^{iI_{\gamma}(\gamma_{s'}, \gamma_0, s')/\bar{h}}, \quad (62)$$

where $C \in \mathbb{C}$ is arbitrary. To the extent that the semiclassical approximation is valid, i.e. that \bar{h} is sufficiently small, this explicitly proves that any significant deviation from a classical path is entirely due to the self field.

Substituting in (13), $G \mapsto G_{\text{sc}}$, $x' \mapsto \gamma_{s'}$ and $x \mapsto \gamma_s$, we first note that one of the β 's, appearing in G_{sc} , connecting $\gamma_{s'}$ with γ_s , must coincide with γ (as γ is a classical path in A , connecting $\gamma_{s'}$ with γ_s). There are, in general, other one-parameter families of *indirect paths*, $s'\beta(\sigma)$, parametrized by s' , connecting $\gamma(s')$ with $\gamma(s)$ not via $\bar{\gamma}$ (e.g. bouncing off of a remote potential). Focusing first on this direct contribution, and using

$$I_{\gamma}(\gamma_s, \gamma_{s'}, s - s') I_{\gamma}(\gamma_{s'}, \gamma_0, s') = I_{\gamma}(\gamma_s, \gamma_0, s) \quad (63)$$

we get

$$\begin{aligned} \phi(\gamma_s, s) &= \frac{\epsilon C}{2} e^{iI_{\gamma}(\gamma_s, \gamma_0, s)/\bar{h}} \int_{-\infty}^{\infty} ds' \mathcal{F}_{\gamma}(\gamma_s, \gamma_{s'}; s - s') \text{sign}(s - s') \mathcal{U}(\epsilon; s - s') \\ &\Rightarrow \phi^{\text{r}}(\gamma_s, s) = \frac{C}{2} e^{iI_{\gamma}(\gamma_s, \gamma_0, s)/\bar{h}} \left[R(s, \epsilon) - \frac{2}{\epsilon} \right], \end{aligned} \quad (64)$$

with

$$R(s) = \int_{-\infty}^{\infty} ds' \mathcal{F}_{\gamma}(\gamma_s, \gamma_{s'}; s - s') \text{sign}(s - s') \mathcal{U}(\epsilon; s - s') \quad (65)$$

¹¹The nature of the approximation involved in the use of the semiclassical propagator can be read from the path integral representation of the propagator. The classical paths dominate that representation and the semiclassical approximation amounts to mis-weighting paths that deviate significantly from classical paths. These, however, enter with a nearly random phase anyway.

some real functional of the EM field and its first derivative (its local neighborhood in an exact analysis) on $\bar{\gamma}$, such that $\lim_{\epsilon \rightarrow 0} [R(s, \epsilon) - 2/\epsilon]$ is finite, implying that (51) is satisfied.

Moving next to the second refined ECD equation, (50), and pushing ∂ into the integral in (13),

$$\begin{aligned} \bar{h}\partial\phi(\gamma_s, s) &= \frac{\epsilon C}{2} e^{iI_\gamma(\gamma_s, \gamma_0, s)/\bar{h}} \int_{-\infty}^{\infty} ds' \left[i\partial_x I_\gamma(x, \gamma_{s'}; s - s') \Big|_{x=\gamma_s} \mathcal{F}_\gamma(\gamma_s, \gamma_{s'}; s - s') \right. \\ &\quad \left. + \bar{h}\partial_x \mathcal{F}_\gamma(x, \gamma_{s'}; s - s') \Big|_{x=\gamma_s} \right] \text{sign}(s - s') \mathcal{U}(\epsilon; s - s'). \end{aligned} \quad (66)$$

The second term in (66) can be neglected for small \bar{h} . Using a relativistic variant of the Hamilton-Jacobi theory we can write

$$\partial_x I_\gamma(\gamma_s, \gamma_{s'}, s - s') = p(s) \equiv \dot{\gamma}_s + qA(\gamma_s)$$

which is independent of s' . The first term in (66) therefore gives

$$\begin{aligned} \bar{h}\partial\phi(\gamma_s, s) &= ip(s)\phi(\gamma_s, s) \quad \Rightarrow \quad \bar{h}\partial\phi^r(\gamma_s, s) = ip(s)\phi^r(\gamma_s, s) \\ \Rightarrow \quad \lim_{\epsilon \rightarrow 0} \text{Re } D\phi^r(\gamma_s, s) f^{R*}(s) &= -\dot{\gamma}_s \lim_{\epsilon \rightarrow 0} \text{Im } \phi^r(\gamma_s, s) f^{R*}(s), \end{aligned} \quad (67)$$

which vanishes by (51), hence (50) is satisfied.

We return now to the contribution of the indirect-paths, β , in the sum over classical paths, appearing in the definition of G_{sc} . The phase of the corresponding integrand in (13) reads

$$\bar{h}^{-1} I_\beta(\gamma_s, \gamma_{s'}, s - s') I_\gamma(\gamma_{s'}, \gamma_0, s'). \quad (68)$$

As distinct $s'\beta$ and $s'\gamma$ see different potentials, and do not lie on the same mass-shell, (68) does depend on s' — the s' -independence, manifested in (63), is a privilege of $\beta = \gamma$. Combined with the smallness of \bar{h} , the contributions of the indirect paths are therefore suppressed by the strong oscillation of the phase (68).

C Conservation of ECD currents

To prove the conservation of the regular current, j^r , defined in (57), we first need the following lemma, whose proof is obtained by direct computation.

Lemma. Let $f(x, s)$ and $g(x, s)$ be any (not necessarily square integrable) two solutions of the homogeneous Schrödinger equation (15), then

$$\frac{\partial}{\partial s}(fg^*) = \partial_\mu \left[\frac{i}{2} (D^\mu f g^* - (D^\mu g)^* f) \right]. \quad (69)$$

This lemma is just a differential manifestation of unitarity of the Schrödinger evolution—hence the divergence.

Turning now to equation (13), written for the rescaled wave-function $\epsilon^{-1}\phi$

$$\phi(x, s) = -2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' G(x, \gamma_{s'}; s - s') f^R(s') \mathcal{U}(\epsilon; s - s'), \quad (70)$$

and its complex conjugate,

$$\phi^*(x, s) = 2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds'' G^*(x, \gamma_{s''}; s - s'') f^{R*}(s'') \mathcal{U}(\epsilon; s - s''), \quad (71)$$

we get by direct differentiation

$$\begin{aligned} q \frac{\partial}{\partial s} & \left[-2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \right. \\ & \quad \left. \mathcal{U}(\epsilon; s - s') G(x, \gamma_{s'}; s - s') \mathcal{U}(\epsilon; s - s'') G^*(x, \gamma_{s''}; s - s'') \right] \\ & = -2q\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\ & \quad \partial_s [G(x, \gamma_{s'}; s - s') G^*(x, \gamma_{s''}; s - s'')] \mathcal{U}(\epsilon; s - s') \mathcal{U}(\epsilon; s - s'') \\ & + [\partial_s \mathcal{U}(\epsilon; s - s') \mathcal{U}(\epsilon; s - s'') + \mathcal{U}(\epsilon; s - s') \partial_s \mathcal{U}(\epsilon; s - s'')] \\ & \quad G(x, \gamma_{s'}; s - s') G^*(x, \gamma_{s''}; s - s''). \end{aligned} \quad (72)$$

Focusing on the first term above, we note that, as G is a homogeneous solution of Schrödinger's equation, we can apply our lemma to that term, which therefore reads

$$\begin{aligned} & -2q\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\ & \partial_\mu \left[\frac{i}{2} (D^\mu G(x, \gamma_{s'}; s - s') G^*(x, \gamma_{s''}; s - s'') - (D^\mu G(x, \gamma_{s''}; s - s''))^* G(x, \gamma_{s'}; s - s')) \right] \\ & \mathcal{U}(\epsilon; s - s') \mathcal{U}(\epsilon; s - s''). \end{aligned} \quad (73)$$

Integrating (72) with respect to s , the left-hand side vanishes (we can safely assume it goes to zero for all x, s', s'' as $|s| \rightarrow \infty$), and the derivative ∂_μ can be pulled out of the triple integral in the first term. The reader can verify that this triple integral is just $\partial_\mu j^\mu$, with j given by (11) and ϕ, ϕ^* are explicated using (70), (71). The regular current, (57), is therefore conserved, provided the s integral over the second term in (72) is missing an ϵ^0 term in its ϵ -expansion.

Let us then show that, in the distributional sense, this is indeed the case. Integrating the second term with respect to s , and using

$\partial_s \mathcal{U}(\epsilon; s - s') = \delta(s - s' - \epsilon) + \delta(s - s' + \epsilon)$, that term reads

$$\begin{aligned}
& -2q\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\
& \mathcal{U}(\epsilon; s' - \epsilon - s'') G(x, \gamma_{s'}; -\epsilon) G^*(x, \gamma_{s''}; s' - \epsilon - s'') \\
& + \mathcal{U}(\epsilon; s' + \epsilon - s'') G(x, \gamma_{s'}; +\epsilon) G^*(x, \gamma_{s''}; s' + \epsilon - s'') \\
& + \mathcal{U}(\epsilon; s'' - \epsilon - s') G(x, \gamma_{s'}; s'' - \epsilon - s') G^*(x, \gamma_{s''}; -\epsilon) \\
& + \mathcal{U}(\epsilon; s'' + \epsilon - s') G(x, \gamma_{s'}; s'' + \epsilon - s') G^*(x, \gamma_{s''}; +\epsilon).
\end{aligned} \tag{74}$$

Using (70) and (71), this becomes

$$\text{Re} -4q\pi^2 \bar{h}^2 i \int_{-\infty}^{\infty} ds' f^R(s') \left[\phi^*(x, s' - \epsilon) G(x, \gamma_{s'}; -\epsilon) + \phi^*(x, s' + \epsilon) G(x, \gamma_{s'}; \epsilon) \right]. \tag{75}$$

Writing $\phi = \epsilon^{-1} \phi^s + \phi^r$ above, and using the short- s propagator (43) plus the explicit form, (46), of ϕ^s , one can obtain the ϵ -expansion of (75). Expanding first $\phi^{s*}(x, s \pm \epsilon)$ in powers of ϵ , the part of the integrand involving ϕ^s can be shown to comprise an ϵ -independent term multiplying $\epsilon^{-2} f_s(\xi^2/2\bar{h}\epsilon)$, with $f_s(y) = \text{sinc}(y) \cos(y) = f_s(-y)$, and another ϵ -independent term multiplying $\epsilon^{-3} f_a(\xi^2/2\bar{h}\epsilon)$, with $f_a(y) = \text{sinc}(y) \sin(y) = -f_a(-y)$. Using the evenness and oddness of f_s and f_a resp., the first term behaves for small ϵ like $\epsilon^{-1} \delta(\xi^2) + O(\epsilon)$, while the second — as $\epsilon^{-1} \delta'(\xi^2) + O(\epsilon)$, both, therefore, do not involve the ϵ^0 coefficient, which is due entirely to ϕ^r . Using (16), the latter's contribution reads in the limit $\epsilon \rightarrow 0$

$$\begin{aligned}
& -8q\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \text{Re} \ i f^R(s) \phi^{r*}(\gamma_s, s) \delta^{(4)}(x - \gamma_s) = \\
& 8q\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \text{Im} \ f^R(s) \phi^{r*}(\gamma_s, s) \delta^{(4)}(x - \gamma_s).
\end{aligned} \tag{76}$$

This is a distribution, supported on $\bar{\gamma}$, which vanishes by virtue of (51). We have therefore shown that $\partial \cdot j^r = 0$ *in the distributional sense*. This is enough to establish the time-independence of the charge, as one only needs to integrate $\partial \cdot j^r = 0$ over a volume in Minkowski's space, and apply Stoke's theorem, to get a conserved quantity. But, in fact, it is easily shown that j^r is a smooth function in the limit $\epsilon \rightarrow 0$, implying a pointwise identity $\partial \cdot j^r = 0$.

To gain a more explicit geometrical insight into the meaning of a 'line sink in Minkowski's space', consider a small space-like three-tube, T , surrounding $\bar{\gamma}$, the construction of which proceeds as follows. Let $\beta(\tau) = \gamma(s(\tau))$ be the world line $\bar{\gamma}$, parametrized by proper time $\tau = \int^s \sqrt{(d\gamma)^2}$, and let $x \mapsto \tau_r$ be the *retarded light-cone map* defined by the relations

$$\eta^2 \equiv (x - \beta_{\tau_r})^2 = 0, \quad \text{and} \quad \eta^0 > 0. \tag{77}$$

Let the 'retarded radius' of x be

$$r = \eta \cdot \dot{\beta}_{\tau_r}. \tag{78}$$

Taking the derivative of (77), treating τ_r as an implicit function of x , and solving for $\partial\tau_r$, we get

$$\partial\tau_r = \frac{\eta}{r} \quad \Rightarrow \quad \partial r = \dot{\beta}_{\tau_r} - \left(1 + \ddot{\beta}_{\tau_r} \cdot \eta\right) \frac{\eta}{r}. \quad (79)$$

The (retarded) three-tube of radius ρ is defined as the space-like three surface

$$T_\rho = \{x \in \mathbb{M} : r(x) = \rho\}.$$

It can be shown in a standard way that the directed surface element normal to $x \in T_\rho$ is

$$d^\mu T_\rho = \partial^\mu r|_{r=\rho} \rho^2 d\tau d\Omega, \quad (80)$$

where $d\Omega$ is the surface element on the two-sphere.

Let Σ_1 and Σ_2 be two time-like surfaces, intersecting T_ρ and T_R . Applying Stoke's theorem to the interior of the three surface composed of T_ρ , T_R , Σ_1 and Σ_2 , and using $\partial \cdot j^r = 0$ there, we get

$$\int_{\Sigma_2} d\Sigma_2 \cdot j^r + \int_{\Sigma_1} d\Sigma_1 \cdot j^r = - \int_{T_\rho} dT_\rho \cdot j^r - \int_{T_R} dT_R \cdot j^r. \quad (81)$$

Realistically assuming that the second term on the r.h.s. of (81) vanishes for $R \rightarrow \infty$, we get that the 'leakage' of the charge, $\int_{\Sigma_2} d\Sigma_2 \cdot j^r - \int_{\Sigma_1} d\Sigma_1 \cdot j^r$, equals to $-\lim_{\rho \rightarrow 0} \int_{T_\rho} dT_\rho \cdot j^r$.

As $dT_\rho = O(\rho^2)$, the leakage only involves the piece of j^r diverging as r^{-2} . This piece, reads

$$\begin{aligned} 2q\bar{h}^2 \int ds \operatorname{Im} \phi^{r*}(x, s) f^R(s) \partial \frac{1}{2\bar{h}\epsilon} \operatorname{sinc} \left(\frac{\xi^2}{2\bar{h}\epsilon} \right) &\xrightarrow{\epsilon \rightarrow 0} 2q\bar{h}^2 \pi \int ds \operatorname{Im} \phi^{r*}(x, s) f^R(s) \partial \delta(\xi^2) \\ &\sim 2q\bar{h}^2 \pi \partial \int ds \operatorname{Im} \phi^{r*}(\gamma_s, s) f^R(s) \delta(\xi^2) = q\bar{h}^2 \pi \sum_{s=s_r, s_a} \operatorname{Im} \phi^{r*}(\gamma_s, s) f^R(s) \partial \frac{1}{|\xi \cdot \dot{\gamma}_s|}, \end{aligned}$$

where $s_r = s(\tau_r)$, and γ_{s_a} is the corresponding *advanced* point on $\bar{\gamma}$, defined by

$$\xi^2 \equiv (x - \gamma_{s_a})^2 = 0, \quad \xi^0 < 0.$$

Focusing first on the contribution of s_r , and using a technique similar to that leading to (79), we get

$$\partial \frac{1}{\xi \cdot \dot{\gamma}_{s_r}} = - \frac{\dot{\gamma}_{s_r}}{(\xi \cdot \dot{\gamma}_{s_r})^2} + \frac{(\dot{\gamma}_{s_r}^2 + \ddot{\gamma}_{s_r} \cdot \xi) \xi}{(\xi \cdot \dot{\gamma}_{s_r})^3} \underset{\xi \rightarrow 0}{\sim} - \frac{\dot{\beta}_{\tau_r}}{mr^2} + \frac{\eta}{mr^3}, \quad (82)$$

where $m = d\tau/ds$ needs not be constant. In the limit $\rho \rightarrow 0$, using $\partial \frac{1}{\xi \cdot \dot{\gamma}_{s_r}} \cdot \partial r|_{r=\rho} \rightarrow m^{-1}$, the contribution of s_r to the flux across T_ρ is most easily computed

$$\begin{aligned} \int_{T_\rho} dT_\rho \cdot j^r &= q\bar{h}^2 \pi \int d\Omega \int d\tau_r m^{-1} \operatorname{Im} \phi^{r*}(\beta_{\tau_r}, \tau_r) f^R(\tau_r) \\ &= 4q\bar{h}^2 \pi^2 \int ds_r \operatorname{Im} \phi^{r*}(\beta_{s_r}, s_r) f^R(s_r). \end{aligned} \quad (83)$$

The contribution of s_a to the flux of j^r is more easily computed across a *different*, (advanced) T_ρ , and gives the same result in the limit $\rho \rightarrow 0$. The fact that ρ can be taken arbitrarily small, in conjunction with the conservation of $j^r(x)$ for $x \notin \bar{\gamma}$, implies that the flux of j^r across *any* three-tube, $T = \partial C$, with C a three-cylinder containing $\bar{\gamma}$, equals twice the value in (83), when C is shrunk to $\bar{\gamma}$. Changing the dummy variable $s_r \mapsto s$ in (83), the formal content of (76) receives a clear meaning using Stoke's theorem

$$\int_C d^4x \partial \cdot j^r = 8q\bar{h}^2\pi^2 \int ds \operatorname{Im} \phi^{r*}(\beta_s, s) f^R(s) \int_C d^4x \delta^{(4)}(x - \gamma_s) = \int_T dT \cdot j^r,$$

which vanishes by virtue of (51).

C.1 Energy-momentum conservation

The conservation of the ECD energy momentum tensor can be established by the same technique used in the previous section. To explore yet another technique, as well as to illustrate the role played by symmetries of ECD in the context of conservation laws, consider the following functional

$$\mathcal{A}[\varphi] = \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \frac{i\bar{h}}{2} (\varphi^* \partial_s \varphi - \partial_s \varphi^* \varphi) - \frac{1}{2} (D^\lambda \varphi)^* D_\lambda \varphi, \quad (84)$$

and let $\phi(x, s)$ be given by (70) for some fixed $A(x)$ and γ_s . Using

$$(i\partial_s - \mathcal{H})\phi = 2\pi^2\bar{h}^2 \left[G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon) \right], \quad (85)$$

directly following from the definition of ϕ , we calculate $\mathcal{A}[\phi + \delta\phi]$ and, after some integrations by parts, we get for the first variation

$$\delta\mathcal{A} = \operatorname{Re} \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x 4\pi^2\bar{h}^2 \left[G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon) \right] \delta\phi^*. \quad (86)$$

Choosing $\delta\phi = \partial\phi \cdot a$, corresponding to $\phi(x, s) \mapsto \phi(x + a, s)$, with infinitesimal $a(x)$, vanishing sufficiently fast for large $|x|$ so as to render $\delta\mathcal{A}$ well defined, we get in a standard way

$$\begin{aligned} \delta\mathcal{A} &= \int_{\mathbb{M}} d^4x (\partial_\nu m^{\nu\mu} - F^\mu{}_\nu j^\nu) a_\mu \stackrel{\text{by eq. (86)}}{=} \\ &= \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \operatorname{Re} 4\pi^2\bar{h}^2 [G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon)] \partial^\mu \phi^*(x, s) a_\mu, \end{aligned} \quad (87)$$

with j given by (11) and

$$m^{\nu\mu} = \int_{-\infty}^{\infty} g^{\nu\mu} \left(\frac{i\bar{h}}{2} (\phi^* \partial_s \phi - \partial_s \phi^* \phi) - \frac{1}{2} (D^\lambda \phi)^* D_\lambda \phi \right) + \frac{1}{2} (D^\nu \phi (D^\mu \phi)^* + \text{c.c.}) ds. \quad (88)$$

(The integrand above, as also the integrands appearing in the definitions of all other ECD currents, can be shown to be a distribution which becomes increasingly focused on the light-cone of γ_s for increasing distance from γ_s .) Writing $\phi = \epsilon^{-1}\phi^s + \phi^r$ in (87), and using the short- s propagator (43) plus the explicit form, (46), of ϕ^s , one can obtain the ϵ -expansion of (87). The regular part of the second line (viz., coefficient of ϵ^0) only involves ϕ^r . In the limit $\epsilon \rightarrow 0$ it reads

$$\begin{aligned} & 8\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \operatorname{Re} f^R(s) \delta^{(4)}(x - \gamma_s) \partial \phi^{r*}(\gamma_s, s) \cdot a(x, s) = \\ & 8\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \operatorname{Re} f^R(s) \partial \phi^{r*}(\gamma_s, s) \cdot a(\gamma_s, s) \end{aligned} \quad (89)$$

which vanishes by virtue of (50) *for any* a . The arbitrariness of a implies that the regular part of the expression in brackets, in the first line of (87), vanishes in the distributional sense,

$$\partial_\nu m^{r\ \nu\mu} - F^\mu_\nu j^{r\ \nu} = 0, \quad (90)$$

with the regular ‘matter e-m tensor’, m^r , defined by the same procedure as j^r , viz. the coefficient of ϵ^0 in its ϵ -expansion. Just like the electric current j^r , the matter e-m tensor can easily be shown to be a smooth function of x , implying pointwise equality in (90). Equation (50), by which (89) vanishes, appears therefore as the condition that no mechanical energy or momentum leak into a sink on $\bar{\gamma}$.

Not surprisingly, m^r is not conserved, due to broken translation covariance induced by $A(x)$. To compensate for this, using Noether’s theorem, we construct an ‘equally non conserved’ radiation e-m tensor, and subtract the two. Consider, then, the following functional of $A(x)$, for fixed ${}^k j^r$, (k labels the different particles)

$$\mathcal{S}[A] = \int_{\mathbb{M}} d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_k {}^k j^r \cdot A. \quad (91)$$

By the Euler Lagrange equations, we get Maxwell’s equations, (4), with $\sum_k {}^k j^r$ as a source. As before, infinitesimally shifting the argument of an extremal A , viz. $A(x) \mapsto A(x+a) \Rightarrow \delta A^\mu = \partial_\nu A^\mu a^\nu$, and following a standard symmetrization procedure of the resultant e-m tensor (adding a conserved chargeless piece $\partial_\lambda (F^{\nu\lambda} A_\mu)$) leads to

$$\partial_\nu \Theta^{\nu\mu} + F^\mu_\nu \sum_k {}^k j^{r\ \nu} = 0, \quad (92)$$

$$\text{with} \quad \Theta^{\nu\mu} = \frac{1}{4} g^{\nu\mu} F^2 + F^{\nu\rho} F_\rho{}^\mu \quad (93)$$

the canonical (viz. symmetric and traceless) ‘radiation e-m tensor’. Summing (90) over the different particles, k , and adding to (92), we get a conserved, symmetric e-m tensor, $\partial_\nu p^{\nu\mu} = 0$, with

$$p = \Theta + \sum_k {}^k m^r. \quad (94)$$

C.2 Charges leaking into world-line sinks

Both methods used above, can be applied to prove the conservation of the regular part of the mass-squared current — the counterpart of (2)

$$b(x) = \int ds B(x, s) \equiv \int ds \operatorname{Re} \bar{h} \partial_s \phi^* D \phi, \quad \text{for } x \notin \bar{\gamma}. \quad (95)$$

In the first method, used to establish the conservation of j^r , the counterpart of (69) is $\partial_s (g^* \mathcal{H} f) = \partial \cdot (\operatorname{Re} \bar{h} \partial_s g^* D f)$, corresponding to the invariance of the Hamiltonian (in the Heisenberg picture) under the Schrödinger evolution. In the variational approach, the conservation follows from the (formal) invariance of (84) $\phi(x, s) \mapsto \phi(x, s + s_0)$. However, the leakage to the sink on $\bar{\gamma}$, between γ_{s_1} and γ_{s_2} , is given by

$$8\pi^2 \bar{h}^3 \int_{s_1}^{s_2} ds \operatorname{Re} \partial_s \phi^{r*}(\gamma_s, s) f^R(s), \quad (96)$$

is not guaranteed to vanish. Note that this leakage (whether positive or negative) is a ‘highly quantum’ phenomenon — proportional to \bar{h}^2 (the term $\partial_s \phi^r$ generally diverges as \bar{h}^{-1}).

Similarly, associated with the formal invariance of (84) under

$$A(x) \mapsto \lambda^{-1} A(\lambda^{-1} x), \quad \phi(x, s) \mapsto \lambda^{-2} \phi(\lambda^{-1} x, \lambda^{-2} s),$$

is a locally conserved dilatation current, the counterpart of the classical current (7),

$$\xi^\mu = p^{\mu\nu} x_\nu - \sum_k 2 \int_{-\infty}^{\infty} ds s {}^k B, \quad \text{with } B \text{ defined in (95)}. \quad (97)$$

The leakage to the sinks on ${}^k \bar{\gamma}$ is due to the second term, involving the mass-squared of the particles. A leakage of mass, therefore, also modifies the scale-charge of a solution.

D Spin- $\frac{1}{2}$ ECD

In a spin- $\frac{1}{2}$ version of ECD, the following modifications are made. The wave-function ϕ is a bispinor (\mathbb{C}^4 -valued), transforming in a Lorentz transformation according to

$$\rho(e^\omega) \phi \equiv e^{-i/4 \sigma_{\mu\nu} \omega^{\mu\nu}} \phi, \quad \text{for } e^\omega \in SO(3, 1), \quad (98)$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, with γ_μ Dirac matrices (not to be confused with γ the trajectory).

The propagator is now a complex, 4×4 matrix, transforming under the adjoint representation, satisfying

$$i \bar{h} \partial_s G(x, x', s) = \left[\mathcal{H} + \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu}(x) \right] G(x, x', s), \quad (99)$$

with the initial condition (16) at $s \rightarrow 0$ reading $\delta^{(4)}(x - x')\delta_{\alpha\beta}$, where $\delta_{\alpha\beta}$ is the identity operator in spinor-space, and g is some dimensionless ‘gyromagnetic’ constant of the theory.

The transition to spin- $\frac{1}{2}$ ECD is rendered easy by the observation that all expressions in scalar ECD are sums of bilinears of the form a^*b , which can be seen as a Lorentz invariant scalar product in \mathbb{C}^1 . Defining an inner product in spinor space (instead of \mathbb{C}^1)

$$(a, b) \equiv a^\dagger \gamma^0 b, \quad (100)$$

with γ^0 the Dirac matrix $\text{diag}(1, 1, -1, -1)$ (again, not to be confused with γ the trajectory) and substituting $a^*b \mapsto (a, b)$ in all bilinears, all the results of scalar ECD are retained. The Lorentz invariance of (100) follows from the Hermiticity of $\sigma^{\mu\nu}$ with respect to that inner product, viz. $(\sigma^{\mu\nu})^\dagger = \gamma^0 \sigma^{\mu\nu} \gamma^0$, and from $(\gamma^0)^2 = 1$.

Let us illustrate this procedure for important cases. By a direct calculation of the short- s propagator of (99), as in section A, the spin can be show to affect the $O(s)$ terms in the expansion of Φ , leading to an equally simple ϕ^s , the counterpart of (46), from which the regular part of all ECD currents can be obtained. The action, (84), from which all conservation laws can be derived, gets an extra spin term

$$\mathcal{A}_s[\varphi] = \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \frac{i\hbar}{2} [(\varphi, \partial_s \varphi) - (\partial_s \varphi, \varphi)] - \frac{1}{2} (D^\lambda \varphi, D_\lambda \varphi) + \frac{g}{2} (\varphi, F_{\lambda\rho} \sigma^{\lambda\rho} \varphi), \quad (101)$$

while the counterpart of the electric current, (11), derived from ϕ , is now a sum of an ‘orbital current’ and a ‘spin current’

$$j^\mu(x) \equiv j^{\text{orb}\mu} + j^{\text{spn}\mu} = \int ds \, q \text{Im}(\phi, D^\mu \phi) - g \partial_\nu (\phi, \sigma^{\nu\mu} \phi), \quad \text{for } x \notin \bar{\gamma}. \quad (102)$$

Expanding (102) in powers of ϵ , the coefficient of ϵ^0 is the regular current, j^r , associated with a particle. Each of the terms composing j^r is individually conserved and gauge invariant. The conservation of the orbital current follows from the $U(1)$ invariance of (101), while conservation of the spin current follows directly from the antisymmetry of σ . This current has an interesting property that its monopole vanishes identically. Calculating in an arbitrary frame, using the antisymmetry of σ , and assuming $j^{\text{spn}i}(x) \rightarrow 0$ for $|\mathbf{x}| \rightarrow \infty$

$$\int d^3\mathbf{x} \, j^{\text{spn}0} = \int d^3\mathbf{x} \int ds \, \partial_0(\phi, \sigma^{00} \phi) - \partial_i(\phi, \sigma^{i0} \phi) = 0 - 0 = 0. \quad (103)$$

The counterpart of (90) becomes (omitting the r identifier, as regularization is implied henceforth)

$$\partial_\nu ({}^k m^{\text{orb}\nu\mu} + g^{\nu\mu} l) = F^\mu{}_\nu \, {}^k j^{\text{orb}\nu} + \frac{g}{2} \int ds \, ({}^k \phi, \sigma^{\lambda\rho} {}^k \phi) \partial^\mu F_{\lambda\rho}, \quad \text{for } x \notin \bar{\gamma}, \quad (104)$$

with m^{orb} the same as (88) with $a^*b \mapsto (a, b)$ in all bilinears, and

$$l(x) = \frac{g}{2} \int ds \, (\phi, F_{\lambda\rho} \sigma^{\lambda\rho} \phi).$$

Note the ‘spin force’ density, vanishing for a constant F , which adds up to the Lorentz force density.

Similarly, adding $\int_{\mathbb{M}} d^4x l(x)$ to the functional in (91), equation (92) becomes

$$\partial_\nu \Theta^{\nu\mu} + \sum_k F^\mu{}_\nu {}^k j^{\text{orb}\nu} + \frac{g}{2} \int ds \left({}^k \phi, \sigma^{\lambda\rho} {}^k \phi \right) \partial^\mu F_{\lambda\rho} + \partial_\nu g \int ds \left({}^k \phi, \sigma^\nu{}_\lambda F^{\lambda\mu} {}^k \phi \right) = 0. \quad (105)$$

Summing (104) over k , and adding to (105), we get the locally conserved e-m tensor

$$\Theta^{\nu\mu} + \sum_k {}^k m^{\text{orb}\nu\mu} + g^{\nu\mu} \eta + g \int ds \left({}^k \phi, \sigma^\nu{}_\lambda F^{\lambda\mu} {}^k \phi \right), \quad x \notin \cup_k {}^k \bar{\gamma}, \quad (106)$$

from which the time-independence of the associated charges follows as in the scalar case, as the extra terms involving spin, do not contain derivatives of ϕ .

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